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Dynamics of exoplanets and exosatellites in binaries.

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Dynamics of exoplanets and exosatellites in binaries.

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Dynamics of exoplanets and exosatellites in binaries.

Ado. Pedro Pablo Campo Díaz

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Santiago de Compostela, a 30 de agosto de 2019

Ado. José Ángel Docobo Durántez

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Dynamics of exoplanets and exosatellites in binaries.

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Dynamics of exoplanets and exosatellites in binaries.

Prof. José Ángel Docobo Durántez, Catedrático de Astronomía do Departamento de Matemática Aplicada da Universidade de Santiago de Compostela

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Na elaboración desta Tese se fixo uso de diferentes catálogos e bases de datos astronómicos. Sobre exoplanetas se consultaron **The Extrasolar Planets Encyclopaedia** (`http://exoplanet.eu/`) e **The Exoplanet Orbit Database** (`http://exoplanets.org/`). En canto a binarias se fixo uso dos diferentes catálogos do **US Naval Observatory** de Washington (`https://www.usno.navy.mil/USNO/astrometry/optical-IR-prod/wds`), do catálogo de órbitas do Observatorio Astronómico Ramón María Aller, **OARMAC** (`http://www.usc.es/astro/catalog.htm`), e do **9th Catalogue of Spectroscopic Binary Orbits** (`http://sb9.astro.ulb.ac.be/`). Tamén temos consultado a base de datos **Simbad**, do Centre de Données astronomiques de Strasbourg (`http://simbad.u-strasbg.fr/simbad/`) e o **Astrophysics Data System** (`https://ui.adsabs.harvard.edu/`).

Esta Tese se desenvolveu ao abeiro dos proxectos de investigación *Estudio de las propiedades astrofísicas y dinámicas de estrellas dobles y múltiples en base a interferometría speckle, fotometría y espectroscopía* (AYA2011-26429), financiado polo Ministerio de Ciencia e Innovación, e *Estudio de binarias cerradas de especial interés dinámico y físico en la era Gaia* (AYA2016-80938-P), financiado polo Ministerio de Economía y Competitividad.

Abstract

Title

Dynamics of exoplanets and exosatellites in binaries.

Short Abstract

This dissertation is the result of the work made under the supervision of J.A. Docobo, Full Professor in Astronomy and Director of the Ramon María Aller Astronomical Observatory of the University of Santiago de Compostela. The core of the dissertation is a compendium of articles published in peer review publications indexed in the Journal Citation Reports, and in the Web of Science. The purpose of this work, suggested by prof. Docobo, is the study of the dynamics planetary systems, focusing on exoplanets and exosatellites in binary stars. The first part of the dissertation is a review of the state of the art in the fields of binary stars and exoplanet research. Then I present the work made in the determination of accurate binary star orbits. The knowledge of precise orbits of these systems is crucial for the determination of the dynamical evolution of the planets in them. The principal part of the dissertation comprises the study of the dynamics of exoplanet systems with exosatellites, and a study of the possible detection of exosatellites by means of the perturbations in the radial velocity signal of the planet.

Abstract

This dissertation includes the work carried out by Pedro Pablo Campo Díaz under the direction of José Ángel Docobo Durántez, Full Professor of Astronomy and Director of the Ramón María Aller Astronomical Observatory (OARMA) of the University of Santiago de Compostela. The core of this work is a compilation of research articles about binary systems, exoplanets, and exosatellites.

First, we include an Introduction in which we review the history and the state of the art in these fields of research, putting them into their scientific context. The investigation of exoplanets (and their satellites) is a natural extension of that realized in multiple stellar systems where, regarding the dynamics, the main change is the ratio of masses. Even at an observational level, many of the techniques that are used for the discovery of extrasolar planets come from the study of binaries, after overcoming the difficulty of the sensitivity of the detectors that previously prevented the observation of such small bodies as planets.

Next, we present a chapter about double stars in which the three types of existing binaries, i. e., visual, spectroscopic, and eclipsing, are discussed. We review the observational techniques that are used for each type and the main methods for the calculation of the orbital elements. In the case of visual binaries, we also deal with the dynamical parallax and its obtention, in addition to presenting an original implementation of the Edwards method for the disentanglement of spectra. In the section about spectroscopic binaries, we talk about the research stay in the Cambridge Observatory which was part of a fruitful collaboration with Professor Roger F. Griffin. There are also specific sections devoted to spectro-interferometric binaries and multiple systems.

The second chapter is focused on the exoplanets and the exosatellites. We begin with their classification and the observational techniques that are used for their discovery, including a discussion of the principal past, present, and future programs as well as the incipient search for exosatellites. After that, we deal with the subject of dynamics, reviewing classical works such as those by Harrington, Dvorak, and others, some of them previous to the discovery of the first exoplanet. We present more recent works, some of them about the stability of exoplanets in binary systems. We also incorporate an original work about the possibility of discovering exosatellites by means of the variation in the radial velocities caused by the perturbations in the orbit of the planet. To that end we use the integration package, TIDES. Finally, we talk about habitability, an exclusive concept belonging to this field of research.

In the last chapter, we include the articles published in high impact international journals about the subjects of this dissertation. There are three articles about binaries, two of them calculating very precise orbits of spectro-interferometric binaries that are framed in the collaboration with Professor R. F. Griffin. The other concerns the study of binaries with measurements concentrated in short arcs of observations, also performing the study of a

system of pre-main sequence stars. In the fourth article, we review the scenarios of four-body systems with exoplanets and exosatellites and conduct a dynamical study of a system with two stars, a planet, and a satellite.

Resumen

En esta tesis se incluye el trabajo realizado por Pedro Pablo Campo Díaz bajo la dirección de José Ángel Docobo Durántez, catedrático en Astronomía y director del Observatorio Astronómico Ramón María Aller (OARMA) de la Universidade de Santiago de Compostela. El nucleo de la tesis está compuesto por una compilación de artículos de investigación sobre sistemas binarios, exoplanetas y exosatélites.

En primer lugar se incluye una Introducción en la que se hace una revisión de la historia y el estado del arte en estos campos de investigación, incluyéndolos dentro de su contexto científico. La investigación en exoplanetas (y sus satélites) es una extensión natural de la realizada en sistemas estelares múltiples, donde con respecto a la dinámica lo que cambia fundamentalmente es la relación de masas. Incluso a nivel observacional muchas de las técnicas utilizadas para el descubrimiento de los planetas extrasolares provienen del estudio de binarias, salvado el impedimento de la sensibilidad de los detectores que en otras épocas impedía la detección de cuerpos pequeños como planetas.

Sigue un capítulo sobre las estrellas dobles, en el que se tratan los tres tipos de binarias existentes, visuales, espectroscópicas y eclipsantes. Se repasan las técnicas de observación usadas en cada tipo y los métodos principales de cálculo de los elementos orbitales. En el caso de las binarias visuales se considera también la paralaje dinámica y su cálculo, además de presentar una implementación original del método de Edwards para la separación de espectros. En la sección sobre binarias espectroscópicas se habla sobre la estancia realizada en el Observatorio de Cambridge, que fue parte de una fructífera colaboración con el profesor Roger F. Griffin. Hay también dos secciones específicas dedicadas a binarias espectro-interferométricas y a sistemas múltiples.

El segundo capítulo está dedicado a los exoplanetas y exosatélites, comenzando por su clasificación y las técnicas de observación utilizadas, incluyendo una revisión de las principales misiones pasadas, presentes y futuras, además de la incipiente búsqueda de exosatélites. Después se trata el tema dinámico, haciendo un repaso de trabajos clásicos como los de Harrington, Dvorak y otros, anteriores incluso al descubrimiento del primer exoplaneta, y de trabajos más actuales, algunos de ellos sobre estabilidad en sistemas binarios. Se desarrolla también un trabajo original sobre la posibilidad de descubrir exosatélites mediante velocidades radiales a partir de las perturbaciones que causan en la órbita del planeta. Para ello se hacen simulaciones utilizando el paquete de integración TIDES. Finalmente se habla sobre habitabilidad, que es un concepto exclusivo de este campo.

En el último capítulo se incluyen los artículos publicados en revistas internacionales de impacto sobre los temas tratados en la tesis. Hay tres artículos sobre binarias, dos de ellos calculando órbitas muy precisas de binarias espectro-interferométricas, que se enmarcan en la colaboración con el profesor R. F. Griffin, y otro sobre el estudio de binarias con medidas

concentradas en arcos cortos de observación, en el que además se hace el estudio de un sistema de estrellas pre-secuencia principal. El cuarto artículo revisa los escenarios de sistemas de cuatro cuerpos con exoplanetas y exosatélites, realizando un estudio dinámico de un sistema con dos estrellas, un planeta y un satélite.

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LIST OF ABBREVIATIONS

ACF	**A**uto**C**orrelation **F**unction
ADI	**A**ngular **D**ifferential **I**maging
ADS	**A**itken **D**ouble **S**tars
AU	**A**stronomical **U**nits
BAO	**B**yurakan **A**strophysical **O**bservatory
BDS	**B**urnham **D**ouble **S**tars
CAHA	**C**entro **A**stronómico **H**ispano-**A**lemán
CCD	**C**harged-**C**oupled **D**evice
CDS	**C**entre de **D**onnées astronomiques de **S**trasbourg
CHARA	**C**enter for **H**igh **A**ngular **R**esolution **A**stronomy
CHEOPS	**CH**aracterising **E**x**OP**lanets **S**atellite
CoRoT	**Co**nvection, **Ro**tation and planetary **T**ransits
E-ELT	**E**uropean **E**xtremely **L**arge **T**elescope
eMCCD	**e**lectron **M**ultiplying **C**harged-**C**oupled **D**evice
ESA	**E**uropean **S**pace **A**gency
ESO	**E**uropean **S**outhern **O**bservatory
ESPRESSO	**E**chelle **SP**ectrograph for **R**ocky **E**xoplanets and **S**table **S**pectroscopic **O**bservations
HARPS	**H**igh **A**ccuracy **R**adial velocity **P**lanet **S**earcher
HATNet	**H**ungarian **A**utomated **T**elescope **Net**work
HIRES	**HI**gh **R**esolution **E**chelle **S**pectrometer)
IAU	**I**nternational **A**stronomical **U**nion
ICCD	**I**ntensified **C**harged-**C**oupled **D**evice
JCR	**J**ournal **C**itation **R**eport
JWST	**J**ames **W**ebb **S**pace **T**elescope
KELT	**K**ilodegree **E**xtremely **L**ittle **T**elescope
LOCI	**L**ocally **O**ptimized **C**ombination (of) **I**mages
MCMC	**M**arkov **C**hain **M**onte **C**arlo
MK	**M**organ-**K**eenan
MLR	**M**ass-**L**uminosity **R**elationship
MOA	**M**icrolensing **O**bservations in **A**strophysics
MNRAS	**M**onthly **N**otices of the **R**oyal **A**stronomical **S**ociety
NASA	**N**ational **A**eronautics and **S**pace **A**dministration
NIRPS	**N**ear **I**nfrared **R**adial velocity **P**lanet **S**earcher

OARMA	**O**bservatorio **A**stronómico **R**amón **M**aría **A**ller
OGLE	**O**ptical **G**ravitational **L**ensing **E**xperiment
PLATO	**PLA**netary **T**ransits and **O**scillations of stars
PSF	**P**oint **S**pread **F**unction
SAO	**S**pecial **A**strophysical **O**bservatory
SB1	**S**ingle-lined **1** **S**pectroscopic **B**inary
SB2	Double-lined **2** **S**pectroscopic **B**inary
SOAR	**SO**uthern **A**strophysical **R**esearch (Telescope)
SOSIE	**S**peckle-**O**ptimize **S**ubstraction for **I**maging **E**xoplanets
SNR	**S**ignal to **N**oise **R**atio
SSDI	**S**imultaneous **S**pectral **D**ifferential **I**maging
SuperWASP	**Super** **W**ide **A**ngle **S**earch for **P**lanets
TESS	**T**ransiting **E**xoplanet **S**urvey **S**atellite
TLOCI	**T**emplate **L**ocally **O**ptimized **C**ombination (of) **I**mages
TDV	**T**ransit **D**uration **V**ariation
TTV	**T**ransit **T**iming **V**ariation
USC	**U**niversidade de **S**antiago de **C**ompostela
VLT	**V**ery **L**arge **T**elescope
VLTI	**V**ery **L**arge **T**elescope **I**nterferometer
WDS	**W**ashington **D**ouble **S**tars

Introduction

In 1992, A. Wolszczan and D. A. Frail (Wolszczan and Frail, 1992) discovered the first planets outside of the Solar System orbiting the pulsar, PSR 1257+12. Three years later, M. Mayor and D. Queloz (Mayor and Queloz, 1995) announced the detection of the first exoplanet in a main sequence star, 51 Pegasi. However, these discoveries were not an absolute surprise. The existence of planets in other star systems had been postulated by Giordano Bruno in the XVI century because he was convinced that the stars were objects similar to the Sun, a hypothesis that was later supported by many astronomers. Despite the certainty that these planets exist, their detection was impossible for many years due to the lack of the resolving power of the measurement devices. Yet, there were announcements of discoveries that were later discredited (Jacob, 1855; van de Kamp, 1969a,b). In 1988, B. Campbell, G. A. H. Walker, and S. Yang (Campbell, Walker, and Yang, 1988), postulated seven stars as candidates to have companions with substellar masses and used their own technique for the measurement of radial velocities (Campbell and Walker, 1979). γ-Cephei was one of them, the only one in which the existence of planets was confirmed later. They discovered that this star was a spectroscopic binary and that its radial velocity curves presented anomalies that they postulated were due to the presence of a giant planet. In 1992, Walker, Yang, and others (Walker et al., 1992) attributed these irregularities to the variability of the main component that was considered to be an orange giant (K0III). It was in 2003, after the reclassification of γ-Cephei as a subgiant (K1IV) and the realization of new high precision radial velocity measurements, when the existence of a planet around said star was confirmed (Hatzes et al., 2003). Since these pioneer discoveries, many more have occurred and there are more than 4000 extrasolar planets at the present time (`http://exoplanet.eu/`, Schneider et al. 2011).

The first years of extrasolar planet research were dominated by the technique of radial velocities, thanks to the development of high resolution spectrographs such as HARPS (High Accuracy Radial velocity Planet Searcher) which is installed in the telescope of 3.6 m. of aperture at the La Silla Observatory, Chile, belonging to the European Southern Observatory (ESO Pepe et al., 2000). Another one is HIRES (High Resolution Echelle Spectrometer) which operates from the 10 m. Keck I telescope at the Keck Observatory, Hawaii, USA (Vogt et al., 1994). In principle, the planets that can be detected with this technique are giant planets similar to Jupiter or even larger, as it was clear with the already commented 51 Pegasi b and γ-Cephei b, and others (Butler and Marcy, 1996; Butler et al., 1997; Marcy and Butler, 1996). However, the improvement in the instrumentation and the extension to the study of very low mass stars has diminished this limit and planets with masses comparable to the

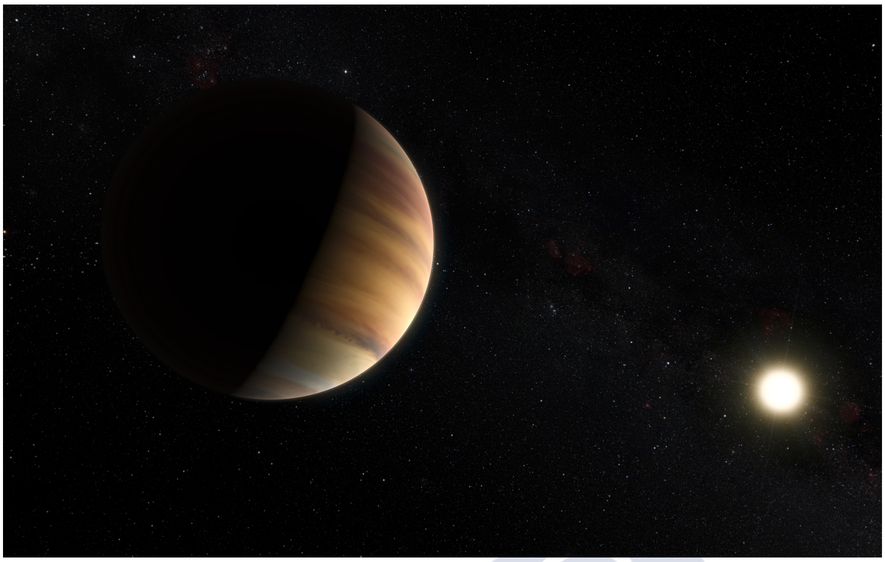

FIGURE 1: Artistic representation of the planet 51 Pegasi b, also known as Dimidium. Image: ESO/M. Kornmesser/Nick Risinger (skysurvey.org)

Earth have been detected with this technique (Anglada-Escudé et al., 2016; Astudillo-Defru et al., 2017a,b; Bonfils et al., 2013; Delfosse et al., 2013; Forveille et al., 2011).

A few years later, the search began for transits in stars with planets discovered by means of radial velocities (Henry et al., 1997) and the first systematic observation programs were carried out (Henry, 1999) which culminated in the first detection of a planet in the star, HD 209458 (Henry et al., 2000). The search using transits soon gained momentum thanks to its advantages, mainly that it was less limited by the size of the planet than the radial velocities. This led to the proposition of space telescopes for the discovery of exoplanets by means of this technique, apart from the land programs.

The first of them, *Convection, Rotation and planetary Transits* (CoRoT), of ESA, had two main goals (*The CoRoT Mission Pre-Launch Status - Stellar Seismology and Planet Finding* 2006): the search for exoplanets by means of transits and the study of asteroseismology which offers information about the inner structure of the stars. CoRoT had a main mirror of 27 cm. of aperture and was operational from 2007 to 2014. It discovered 32 planets with a list of 600 candidates to be confirmed. The first planet discovered by this telescope, CoRoT-1-b, is a hot Jupiter and it was the first case in which the phase of the planet could be observed in the visual range (Barge et al., 2008). Among its detections, we can also highlight CoRoT-7-b, a superEarth with a diameter of 1.58 times that of our planet and, at that time, the smallest detected in a main sequence star (Léger et al., 2009).

The next space mission, Kepler of NASA, has been the most successful program until now in the search for extrasolar planets (Borucki et al., 2010). With an aperture of 0.95 m., Kepler was launched in 2009 and worked until 2018. The original mission objective was the study

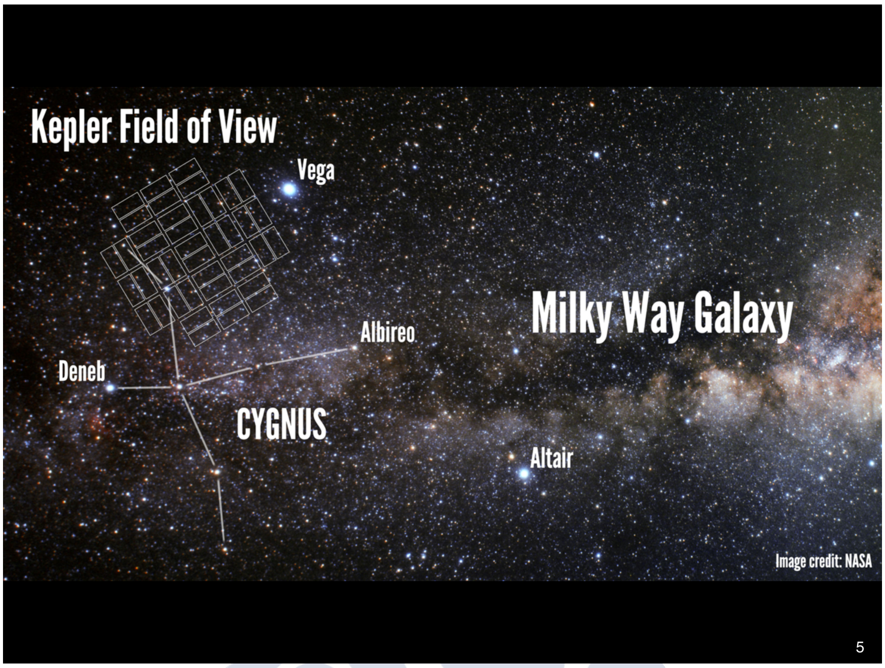

FIGURE 2: Field of view of the Kepler space telescope. Image: NASA

of an area of the sky among the constellations of Lyra, Cygnus, and Draco, away from the ecliptic to avoid the entrance of sunlight in the main mirror. The failure of two of its reaction wheels that were used for pointing the telescope led to the beginning of the K2 phase of the mission in which it observed several zones of the ecliptic. In total, between the original mission and the K2 extension, this telescope discovered 2734 confirmed planets with a list of 2951 candidates. One of its discoveries was Kepler-22-b which, with a diameter of 2.4 times that of our planet, was the first superEarth located in the habitability zone of its star. Kepler has detected a great number of planets of a size comparable to the Earth, or even smaller, such as Kepler-37-b which is a little larger than the Moon. The success of these space missions motivated the planification of their successors. One of them, TESS (NASA) is already in orbit, and the others, CHEOPS and PLATO (ESA), are in development.

Despite the difficulty, we began to observe planets by imaging. Most of the detections listed in the catalogs correspond to objects with a mass in the range of brown dwarfs (see for example Bihain et al., 2010; Nakajima et al., 1995; Rebolo et al., 1998), although some of them correspond to planetary mass objects (Zapatero Osorio et al., 2000). Among them, we find Fomalhaut b (Kalas, Graham, and Clampin, 2005) that is the least massive and that was detected with an estimated mass lower than 3 $\mathcal{M}_{Jup}$. The same limitations appear in the case of astrometry, with most detections in the range of brown dwarfs, including the object postulated by the team of OARMA and others in the Gliese 22 system (Docobo et al., 2008a). The smallest is HD 176051 b (Muterspaugh et al., 2010) with a mass of 1.5 $\mathcal{M}_{Jup}$.

Another fruitful technique despite its drawbacks is gravitational microlensing. There are two main programs that use it, the *Optical Gravitational Lensing Experiment* (OGLE Udalski et al., 1992), a cooperation among the Carnegie Institution of Washington, Princeton University, and the Warsaw University Observatory, which consists of a 1 m. telescope located at the Las Campanas Observatory (Chile) and the *Microlensing Observations in Astrophysics* (MOA Muraki et al., 1999), a Japan and New Zealand collaboration that operates from the University of Canterbury Mt. John Observatory in New Zealand, with a 1.8 m .telescope.

Most of these discoveries were carried out with techniques used in the study of double stars, mainly radial velocities and transits (corresponding to spectroscopic and eclipsing binaries, respectively), as well as imaging and astrometry (visual binaries). From a dynamical point of view, the study of exoplanets in binary and multiple systems is much more interesting. That is why these fields, double and multiple stars as well as extrasolar planets, are closely related.

Double stars, along with the work fields related to their study, constitute a fundamental subject of research in Astronomy. In the XVII century, it was discovered that the star, Mizar, in the Ursa Major constellation, had a close companion apart from the pair that it forms with Alcor which is visible with the naked eye. Mizar was observed as a double star by G. B. Riccioli around 1650 (Riccioli, 1651) although some of the letters that are preserved in the National Library in Florence indicate that it had already been seen as a double by B. Castelli and G. Galilei in 1617 (Fedele, 1949). During the XVII and XVIII centuries, other pairs of stars were observed for the first time, such as α-Centauri, α-Geminorum (Castor), or γ-Virginis. The increasing number led J. Mitchell to postulate that they weren't random alignments but physical systems (Heintz, 1978). However, it was W. Herschel who started to systematically follow pairs of close stars at the end of the XVIII century. His goal was to observe the annual paralactic movement in order to measure the parallax and, therefore, their distance to us. He was not able to achieve his objective but this practice led him to discover the orbital motion of double stars (Herschel, 1803). The distance to a star was determined for the first time a few decades later by F. W. Bessel who, in 1838, estimated the parallax of the star, 61-Cygni, to be 0”3136 (which is equivalent to a distance of 3.19 parsecs), by comparing its position with another six stars (Heintz, 1978). Curiously, this was a binary star from the list of Herschel.

Also around that time, J. Goodricke proposed two models to explain the variability of the star, Algol in Perseus which was already known since antiquity (Goodricke, 1783). He suggested that this phenomenon might be caused either by spots similar to those on the photosphere of the Sun or by the transit of a giant planet. However, these explanations were not accepted because of the size that the planet or the spots should have in order to produce the observed variations in brightness. The astronomers at that time noticed another possibility that there was another star in the system which, when eclipsing (and being eclipsed by) the other component, diminished the measured brightness of the system.

At the beginning of the XIX century, F. G. W. Struve began to measure double stars by using a filar micrometer (Struve, 1837, 1852), a variation of the device invented by W.

FIGURE 3: Portrait of William Herschel. Image: public domain

Gascoigne around 1639 (Towenley, 1666). The pioneering works by W. Herschel and F. G. W. Struve were initially continued by their respective sons, J. Herschel and O. Struve (Heintz, 1978). However, F. Savary was the first person to calculate the orbit of a binary star, ξ-Ursae Majoris (Savary, 1827) a few years ahead of J. Herschel (Herschel, 1833). Bessel attributed the variations observed in the proper motions of Sirius and Procyon to the presence of companion stars (Bessel, 1844) which was the first discovery of components of a system by means of astrometric techniques (without direct observation). Sirius B was first observed by A. G. Clark in 1862 whereas Procyon B was seen by J. M. Schaeberle in 1896 (Heintz, 1978)

The discovery of the Doppler-Fizeau effect halfway through the XIX century cleared the way for the third technique for the study of double stars (Heintz, 1978), along with direct observation and the study of eclipses. Thanks to this well known physical effect, it is possible to determine the radial component of the orbital velocity of a star in a binary system through the displacement of the absorption lines in its spectrum. After a first attempt (Huggins, 1868), E. C. Pickering was able to observe (Pickering, 1890) the periodic duplicity of the spectral lines in Mizar due to the orbital motion. Many other discoveries followed, including the confirmation of the binarity of Algol (Vogel, 1890). This type of binary is called spectroscopic and they are, in turn, divided in two subtypes. If the spectral lines of both components can be seen in the spectrum, they are called double-lined spectroscopic

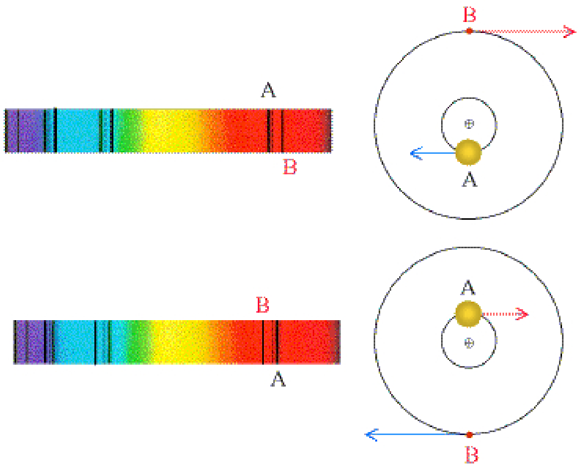

FIGURE 4: The Doppler effect in binaries. Image: Margaret Murray Hanson

binaries (SB2) whereas, if only the lines of the brightest component appear, they are known as single-lined spectroscopic binaries (SB1).

Regarding the visual observations, in 1906, S. W. Burnham published his catalog of double stars (BDS) with more than 13000 entries that were based mainly on observations carried out at the Lick and Yerkes Observatories (Burnham, 1906). This catalog was incorporated into the Catalog of Double Stars by R. S. Aitken (ADS) in 1926, with more than 17000 stars, along with measurements by Doolittle and by himself (Aitken, 1926). During the XX century, a large number of observers significantly increased the number of known binaries and micrometer measurements, with thousands of observations. Some of them were, for example, P. Baize, G. van Biesbroeck, W. H. van den Bos, P. Couteau, W. D. Heintz, R. T. A. Innes, G. P. Kuiper, P. Muller, R. A. Rossiter, G. A. Starikova, R. H. Wilson, and C. E. Worley, and many others (Docobo, 2016).

Around that time, the work on binaries began in Spain. J. Comas Solá, among his multiple activities, performed measurements of a large number of double stars even before becoming the Director of the Fabra Observatory. He first worked with R. Patxot and later he worked alone, and he discovered a new pair, SOL1 (Comas Solá, 1898, 1899, 1900, 1902). However, R. M. Aller was the true pioneer in this field in Spain. In addition to the large number of micrometer measurements carried out both at Lalín and at Santiago de Compostela (Aller, 1930, 1934, 1936), he calculated the first orbits in our country (Aller, 1935, 1939).

In 1887, A. A. Michelson and E. W. Morley published their famous experiment about the speed of light (Michelson and Morley, 1887). As early as 1900, J. M. Barr proposed the use of the Michelson interferometer for the study of the multiple star, Capella (Barr, 1900), although said study was delayed until 1920 (Michelson, 1920b,a). These works initiated the use of interferometric techniques for the study of visual double stars. J. A. Anderson published the interferometric measurements of several pairs (Anderson, 1920a,b), followed several years

Figure 5: Portrait of Ramón María Aller

later by P. W. Merril (Merrill, 1922). The next milestone is attibuted to F. W. Finsen who, in 1951, developed an eyepiece interferometer (Finsen, 1951) that he used to conduct a large number of observations of close pairs at the Johannesbourg Observatory. However, interferometry reached its maturity in 1970 due to the development of the so-called "speckle interferometry" by A. Labeyrie (Labeyrie, 1970). This technique permits the attenuation of the effect of the atmosphere in order to reach the diffraction limit of the telescope, thereby allowing the observation of very close pairs. Many authors have used this technique in the last decades, attaining an unprecedented precision in the measurement of the relative positions. Some of them are H. A. McAlister, Y. Balega, W. I. Hartkopf, A. Tokovinin, E. P. Horch, B. D. Mason, J. L. Prieur, M. Scardia, G. Weigelt, as well as the research group of OARMA directed by J. A. Docobo. A Ph.D. dissertation elaborated by J. Gómez and directed by Professor Docobo was recently presented, containing a description of the work carried out at the OARMA in this field (Gómez, 2019).

The study of systems that can be detected by means of different techniques is especially interesting because they provide more astrophysical information. The study of binaries that are both spectroscopic and eclipsing is customary, as these techniques favor short period systems. However, our group focuses on the study of visual binaries, therefore it is our preference to research spectro-interferometric systems, with both visual (obtained from interferometric measurements) and spectroscopic orbits that are calculated from their radial velocities. As we will see in the chapter corresponding to double stars, this type of system permits the obtention of the individual masses of the components as well as the orbital parallax which is an independent test for the data obtained by astrometric missions such as *Hipparcos* or *Gaia*. From the beginning of the interferometric observations, our interest in these systems was established and many authors started to obtain combined orbits (Balega, Bonneau, and Foy, 1984; Balega and Ryadchenko, 1984; Bonneau et al., 1986; McAlister, 1976, 1977, 1978). R. F. Griffin of the Cambridge Observatory contributed largely to this effort. He has published more than 260 articles about spectroscopic orbits and many of those systems have also been observed by means of interferometry. In the last years, our team at OARMA has published several works in this field (Docobo et al., 2014b, 2017a, 2018a,b), two of them in collaboration with Professor Griffin that have been included in this dissertation. We have also published a methodology to calculate the tridimensional orbit of a spectroscopic binary by using one visual observation (preferentially a high resolution measurement) and the parallax (Docobo et al., 2014a). The Ph.D. dissertation by A. Abushattal, also conducted under the supervision of J. A. Docobo at OARMA (Abushattal, 2017), collected different techniques to work with spectroscopic binaries, some original methodologies included, as well as a compilation of the works by R. F. Griffin and OARMA until that time.

Chapter I

Double stars

As it was established in the Introduction of this Memory, the study of binaries and, in general, of double stars (today the term “binary” is reserved preferentially for the closest cases), has a great significance for obtaining the fundamental physical parameters of the stars and thus being able to formulate precise models of their behavior and evolution. Double stars are classified according to the technique with which their binarity is detected and further study is conducted. There are three types, visual, if it is possible to observe both components by means of optical instrumentation (e.g. speckle interferometry); spectroscopic, if we can measure the radial velocity caused by the Doppler-Fizeau effect when the components of the system revolve around the orbit; and eclipsing, if our line of sight is contained (or close to) the orbital plane, so that the components undergo mutual periodic eclipses. We can also consider astrometric binaries as a particular case of the visual. They are systems in which it is only possible to observe the brighter star and its movement with respect to other neighboring stars due to the difference in magnitude. This classification is not exclusive and there may be systems that belong to two or even three of the types. These stars are the most interesting because they provide more astrophysical information.

At OARMA, the main field of research is visual double stars, following the school that Ramón María Aller established in the USC (Aller, 1943, 1957; Docobo, 2011, 2016). However, due to the advances in the observational devices, we work with more and more spectro-interferometric stars. In this chapter we review the techniques that are used for the study of visual double stars and, more succinctly, of spectroscopic and eclipsing binaries. We discuss the algorithm proposed by P. Baize and L. Romani (Baize and Romani, 1946; Heintz, 1978) for the determination of the dynamical parallaxes and we focus on the Edwards algorithm which allows us to obtain the individual spectra of a double star from the combined spectrum and the difference in magnitude between the components. We will present a novel formulation of the algorithm that we designed and that generalizes the original process by using the bolometric corrections. The final section is a summary of the state of the art in multiple star research.

I.1 Visual binaries

I.1.1 Observation

The first step in the study of visual double (Docobo, 2002, 2016) stars is observation in order to be able to calculate their orbits. The orbit, along with the parallax (π), permits us to determine the physical parameters of the stars, mainly the sum of the masses of the components or, in some instances, even the individual masses. Other physical data can be obtained from the observations, for example, the difference in magnitude between the components, Δm. During an observation, we measure the position of the faintest component (secondary) with respect to the brightest component (primary) and, if they have the same magnitude, the star with the largest right ascension is chosen as the primary. We will see the projection of both components on the plane perpendicular to our line of sight and, as we are observing objects at huge distances, that projection can be considered to be cylindrical. In each observation, we obtain a triplet of data $(\theta, \rho; t)$, where t represents the time of the observation, usually given in the Besselian epoch (B1950.0), although the International Astronomical Union (IAU) Commission G1, which is in charge of the study of double and multiple stars, has stated the necessity of changing to the use of the Julian epoch (J2000.0) in accordance with the recommendations of the IAU General Assembly celebrated in Grenoble in 1976 (Aoki et al., 1983). θ stands for the angle between the North direction and the radio vector that joins the primary and the secondary components. This angle is measured in degrees and follows the N-E-S-W path (see Figure I.1). Finally, ρ indicates the angular separation between both stars and it is measured in arcseconds although, when observing closer systems, it is necessary to use miliarcseconds.

Several procedures have been used throughout history in order to obtain the measurements (θ, ρ). The main instrument before speckle interferometry was the filar micrometer. Prior to the introduction of the micrometer by F. G. W. Struve, W. Herschel, and others already performed observations of double stars, although with lower precision. The simplest design of a micrometer (de Villiers, 1999) consists of a part that is placed between the eyepiece and the telescope, with two thin and resistant filaments (they were formerly made of threads from spider nests, today of synthetic fibers) forming a crosshair and another thread parallel to one of the other. The observation begins by fixing the position of the primary star in the center of the crosshair, and then the micrometer is rotated until the secondary is placed on the fixed thread perpendicular to the mobile one. This movement measures the position angle with respect to the North direction by means of a graduated circle in the micrometer. Next, the mobile thread is displaced onto the secondary star by means of a graduated screw which permits the measurement of the angular separation (see Figure I.2). Usually, the telescope is moved to center the secondary in the crosshair and the mobile thread is placed over the primary star. The mean of both measurements is taken as the final value in order to avoid index errors.

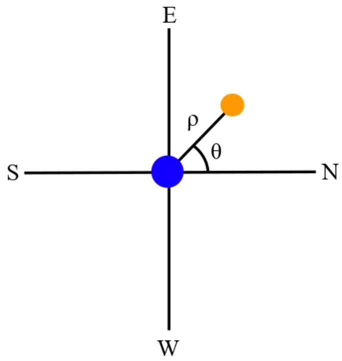

FIGURE I.1: position angle, θ, and angular separation, ρ, of a binary system. The blue and orange circles represent the primary and the secondary components, respectively.

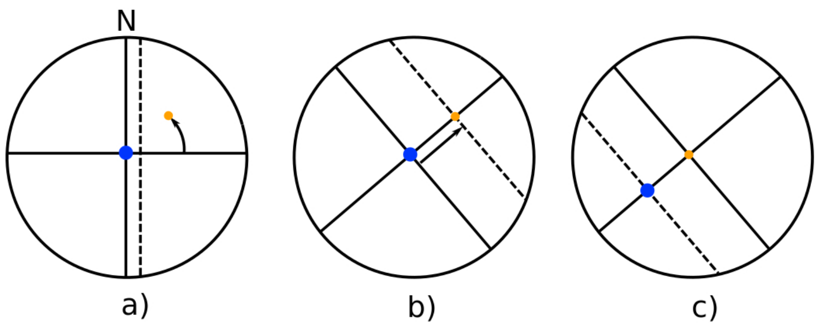

FIGURE I.2: Schematic of a filar micrometer and steps for a micrometric measurement of θ and ρ. The blue and orange circles represent the primary and the secondary components, respectively. The solid perpedicular lines stand for the crosshair of the micrometer, and the dashed line indicate the mobile thread.

Despite their simplicity, these devices have two downsides that led to the adoption of the modern digital methods. On one hand, this technique is heavily dependent on the experience of the observer and it requires a lot of practice to master the positioning of the threads. On the other hand, it is largely affected by the atmospheric conditions and a bad seeing can cause large errors in a measurement.

Many observations were conducted using photographic plates (see for example Guntzel-Lingner, 1962; Güntzel-Lingner, 1962; Hertzsprung, 1917, 1919, 1920, 1940, 1942b; Hertzsprung and Albada, 1958; Thiele, 1903, 1907) although the development of photography could not surpass the micrometer as the preferred tool in the double star studies, at least not before the arrival of digital photography. This technique was limited by observer errors, measurement errors, and errors inherent to the image, in addition to the seeing conditions (Heintz, 1978; Hertzsprung, 1942a). All of these errors made it very difficult to achieve angular separations lower than 1″ . This excluded the systems with shorter periods which had a wider arc of observations and permitted toÂ¡he determineÂ¡ation of more precise orbits. The introduction of digital photography overcame these difficulties, mainly with the inclusion of CCD (Coupled Charged Device) chips, and the use of the micrometers has been gradually abandoned.

The development of high resolution techniques in the last decades caused a qualitative leap in the research of visual double stars. The purpose of these techniques is to eliminate (or at least largely reduce) the effect of Earth's atmosphere in order to attain the diffraction limit of the telescope which is given by the formula:

$$\rho = 1.22\frac{\lambda}{D}, \tag{I.1}$$

where ρ indicates the minimum separation attainable by a telescope with D aperture, while observing in the λ wavelength. The wavefront that reaches the telescope can be considered to be plane due to the large distance towards the astronomical sources. When observing a point source of light with the telescope in absence of atmospherical turbulence, the object does not appear as a point in the plane of the image; the different parts of the objective emit wavefronts that produce an interference pattern called Point Spread Function (PSF) when they overlap in the focus. For a circular or annular aperture, as is the case of most telescopes, the shape of the PSF becomes a bright central disc with concentric rings around it and it is called the Airy function (or Airy disc). The center of the disc corresponds to the maximum of this Airy function, whereas the minima appear in the separation between the disc and the first ring, and between consecutive rings, due to the destructive interference between the different wavefronts. The Rayleigh criterion points to the distance between the maximum and the first minimum and, in the case of two point sources with that separation, the maxima of their PSF would overlap with the first minimum of the other PSF, and the images can be separated (see Figure I.3).

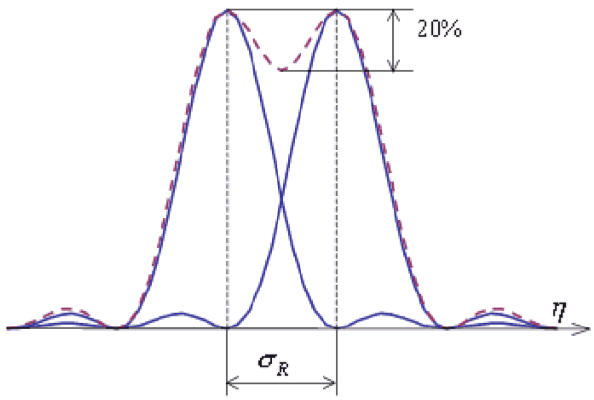

FIGURE I.3: The Rayleigh criterion. The maximum of the PSF of each point source matches the first minimum of the PSF of the other source. Image: Prudyus et al. (2017)

This ideal situation is not found in practice. The wavefront is distorted when passing through the atmosphere and the PSF is not the Airy disc but the instant image of the stars is a variable pattern of speckles. For a more prolonged exposition, the PSF obtained is a fuzzy disc that is called the "seeing disc". The diameter of the seeing disc increases with the turbulence and, in any case, it is always larger than the diameter of the Airy disc, thus the real resolution never attains the theoretical limit of the telescope. The atmospheric conditions may vary rapidly, sometimes in minutes, and these limitations are not easy to overcome.

Several techniques have been developed to minimize the effect of the atmospheric turbulence. The first of them is *speckle interferometry*. This procedure consists of taking a large number of images with short expositions (on the order of miliseconds) and combining them by means of Fourier analysis after correcting the possible offsets among them (using a method called shift-and-add). The images obtained in this way are practically instantaneous and the speckle pattern becomes visible. The fundamental hypothesis of the work of Labeyrie is that the minimum size of the speckles matches the size of the theoretical Airy disc of the star. If we call of the object $O(\alpha, \beta)$, and the intensity distribution of the image $I(\alpha, \beta)$, we can write the following equation:

$$I(\alpha, \beta) = O(\alpha, \beta) \otimes |p(\alpha, \beta)|^2, \tag{I.2}$$

where $p(\alpha, \beta)$ is the Fourier transform of the perturbed pupil of the telescope, $P(x, y)$, therefore the equation I.2 shows how the telescope deforms the real image of the object, O, to obtain the captured image, I, under certain seeing conditions. If we apply Fourier transforms to both sides of the equation, and square them, we obtain:

$$|i(x,y)|^2 = |o(x,y)|^2 |\mathcal{A}(P(x,y))|^2. \tag{I.3}$$

$\mathcal{A}(P(x,y))$ is the autocorrelation function of $P(x,y)$. This function is not known at first, although its mean value can be estimated from the observations and, with it, we can calculate the amplitude of the Fourier transform of the object. A combination of many images yields a better signal-to-noise ratio (SNR), therefore the equation I.3 becomes:

$$\sum |i(x,y)|^2 = |o(x,y)|^2 | \sum \mathcal{A}(P(x,y))|^2. \tag{I.4}$$

In principle, we could not reconstruct the image because we do not know the phase but due to the central simmetry of the stars, this is not a problem. When we work with double stars, the combination of the obtained images yields an interference pattern with parallel fringes with constant separation. The distance between these fringes can be used to measure the separation between the components and the contrast, which is defined from the maximum and minimum intensities on the fringes, I_{max} and I_{min}, as follows:

$$K = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}, \tag{I.5}$$

permits the calculation of the difference in magnitude between the components by using the following equation (Gezari, Labeyrie, and Stachnik, 1972):

$$\Delta m = 2.5 log \frac{1 + \sqrt{1 - K^2}}{1 - \sqrt{1 + K^2}}. \tag{I.6}$$

Besides the *speckle interferometry*, another technique to overcome the limitation of the atmosphere is *adaptive optics*. In order to use it, the telescope must have a deformable mirror so that its surface can be adjusted to correct the distortions of the wavefront. This technique cannot be implemented with a telescope with a single-piece mirror but it is possible in large telescopes with segmented mirrors in which each tile has a drive that permits the modification of its position. Although H. W. Babcock (Babcock, 1953) already conceived the idea of adaptive optics around the middle of the XX Century, it could not be implemented until much later when the technical advances, mainly in the field of computing, made it feasible (see for example Beckers, 1993, and references therein). The biggest difficulty comes from the evaluation of the deformations of the wavefront which is computationally costly. In order to evaluate it, it is necessary to observe a reference object which is usually a bright star (guide star). However, this guide star must be close to the object that we want to observe so that the measured distortion is coherent and, on many occasions, this is not the case. That is why many adaptive optics telescopes use a laser to create a synthetic reference (laser beacon or laser guide star).

FIGURE I.4: Laser beacon shot by the ESO's VLT for its adaptive optics system.
Image: ESO, Y. Beletsky

A more simple alternative is the use of *lucky imaging* (Fried, 1978). This technique is based on the principle that, for a short enough exposure time, the atmosphere is stationary. A high-velocity camera is used with very short exposures (on the order of miliseconds). The best images are selected (usually around 10% of them), and they are combined by means of a shift-and-add method. Unlike *speckle interferometry*, the images are added directly without applying Fourier analysis. This technique is less effective for telescopes with larger apertures, because the likelihood of obtaining low distortion images is lower. It is considered to be inefficient for apertures larger than 3 meters, at least with the data acquisition speed of modern cameras.

Finally, we have *aperture synthesis interferometry*. This technique consists of combining pairs the images coming from several telescopes. Each pair of telescopes in an interferometer forms a baseline. The observations made with many baselines, or with a few of them but varying with the Earth's rotation, can be used by means of the Fourier analysis to reconstruct the image with a resolution corresponding to a telescope with an apeture equivalent to the longest baseline (Burke and Graham-Smith, 2009). Aperture synthesis interferometers are widely used in radioastronomy, even with planetary-scale baselines (Doeleman et al., 2009), which is called very long baseline interferometry (VLBI). They are less common for shorter wavelegths because the atmospheric scintillation and the required infraestructure stability increase with frequency, nevertheless, there are several optical and infrared interferometers. Among them, we may highlight the VLTI (Very Large Telescope Interferometer) from the European Southern Observatory (ESO), located in Cerro Paranal (Chile) with four telescopes with apertures of 8.2 m. and four auxiliary 1.8 m. telescopes; the Keck Observatory in Mauna Kea (Hawaii, USA) with two 10 m. telescopes; or the interferometer of the Center for High Angular Resolution Astronomy (CHARA array) at Georgia State University, located in Mount Wilson (California, EEUU).

At OARMA, after the recovery of the center in 1981 under the direction of professor José

Ángel Docobo, several campaigns of micrometric observations of double stars have been conducted at the Observatories of Santiago de Compostela, Fabra (Barcelona), Niza and Pic du Midi (Francia), as well as with the 1.52 m. telescope from the Centro Astronómico Hispano-Alemán of Calar Alto (CAHA, Almería), which have resulted in several publications: Couteau, Docobo, and Ling (1993), Couteau and Ling (1988, 1991), Couteau et al. (1989), Docobo (1986, 1989, 1998), Docobo, Costa, and Ling (1984), Docobo and Ling (1994), Docobo, Ling, and Lanchares (1999), Docobo and Prieto (1993), Docobo et al. (1991), Ling and Couteau (1992), Ling (1987), Ling and Prieto (1997, 1998, 2000), and Ling and Lanchares (1993). In addition to this, OARMA made a significant effort to update the instrumentation by means of the incorporation of an ICCD speckle interferometry camera (Andrade and Docobo, 2006a; Docobo et al., 2001a,b, 2004, 2007a,b,c, 2008b; Docobo et al., 2006; Hartkopf et al., 2000; Tamazian et al., 2000; Tamazian and Docobo, 2006) and an eMCCD speckle interferometry camera (Docobo et al., 2017b; Docobo et al., 2010, 2019; Gomez et al., 2016; Tamazian et al., 2011). The OARMA research group obtained speckle measurements at the CAHA (1.52 and 3.5 m. telescopes), Mount Wilson (EEUU, 2.5 m.), the Byurakan Astrophysical Observatory (BAO, Armenia, 2.6 m.), the Special Astrophysical Observatory (SAO, Rusia, 6 m.), and the Southern Astrophysical Research (SOAR, Chile, 4.2 m.), and CCD resgistries at the Observatorio de Llano del Hato (Abad, Docobo, and della Prugna, 1998), as well as at CAHA (Abad et al., 2004; Docobo et al., 2000). The contribution of OARMA to binary star research is also demonstrated by the organization of several international meetings: "International Workshop. Visual Double Stars: Formation, Dynamics and Evolutionary Tracks" (1996), "International Workshop. Double and Multiple Stars: Dynamics, Physics, and Instrumentation" (2009) and "International Workshop. Binaries Inside and Outside the Local Interstellar Bubble" (2011). We have to add the edition of the Information Circular of the Commission 26 (nowadays Comission G1) of the International Astronomical Union (IAU) since 1993 (n^{er} 149). All of these factors represent an important contribution of the OARMA personnel within the IAU. Professor Docobo was elected as Vice President of Commission 26 between 2006 and 2009, and occupied the Presidency from 2009 to 2012.

I.1.2 Orbit calculation

In order to obtain the astrophysical parameters of a binary, the next step is orbit calculation (Abad, Docobo, and Elipe, 2017; Docobo, 2016). Whenever both components are sufficiently separated, the orbital movement of double stars can be modelled using the laws of movement within the framework of Newtonian gravity, i. e., by means of the equation of the classic two-body problem:

$$\ddot{\vec{r}} = -\mathcal{G}(\mathcal{M}_1 + \mathcal{M}_2)\frac{\vec{r}}{r^3}, \tag{I.7}$$

where $\vec{r}$ is the vector joining both stars, r represents its module, $\mathcal{M}_1$ and $\mathcal{M}_2$ stand for the masses of the components, and $\mathcal{G}$ denotes the gravity constant. This approximation is satisfactory in most cases although, in close systems, it may be neccessary to include other terms in the equation to model non-spherical shapes, relativistic corrections, radiation pressure, etc. In systems with massive and/or evolved components, there may be strong mass-loss phenomena (Andrade, 2007).

The equation I.7 is well known and it has a general solution. As it has seven degrees of freedom (six, if we know the sum of the masses as for example, in the Solar system), we need seven parameters to describe the solution. Two bodies that are moving following this equation will describe curves that are conic sections with respect to the center of masses of the system (absolute orbit). If we consider only periodic orbits, both components will describe ellipses with the center of masses in one of the foci. If we change the reference frame and we move the origin to the main component of the system, we obtain an equivalent result in which the secondary describes an elliptic orbit with the primary in one of the foci that is called the relative orbit. Moreover, this orbit follows Kepler's Laws, and the radius-vector between the components describes equal areas in equal times and it verifies:

$$\vec{r} \wedge \dot{\vec{r}} = 2\vec{C} \ (constante), \tag{I.8}$$

The orbits that are the solution of this problem are known as Keplerian orbits and they can be described by means of the seven Campbell orbital elements:

- P, period of the orbit.
- T, time of passage through the periastron, the point at which both bodies are closest.
- e, eccentricity of the ellipse.
- a, semimajor axis of the ellipse.
- I, inclination with respect to a reference plane.
- Ω, angle of position of the ascending node.
- ω, argument of the periastron.

The reference plane that is used is the plane perpendicular to the line of sight, passing through the main star. More concretely, in binary star research, the reference frame that is used is left-handed, Cartesian, and orthogonal, with the X axis in the North (N) direction, the Y axis in the East (E) direction, and the Z axis passing through the main star and going in the opposite direction with respect to the observer. The reference plane previously mentioned is, therefore, the OXY plane in this system (see Figure I.5). In a left-handed reference frame like this, the inclination, I, is defined as the angle between the angular moment vector, $\vec{C} = \frac{1}{2}\vec{r} \wedge \dot{\vec{r}}$ and the Z^- axis (in the direction of the Earth), so that for direct motion (clockwise), $I < 90°$,

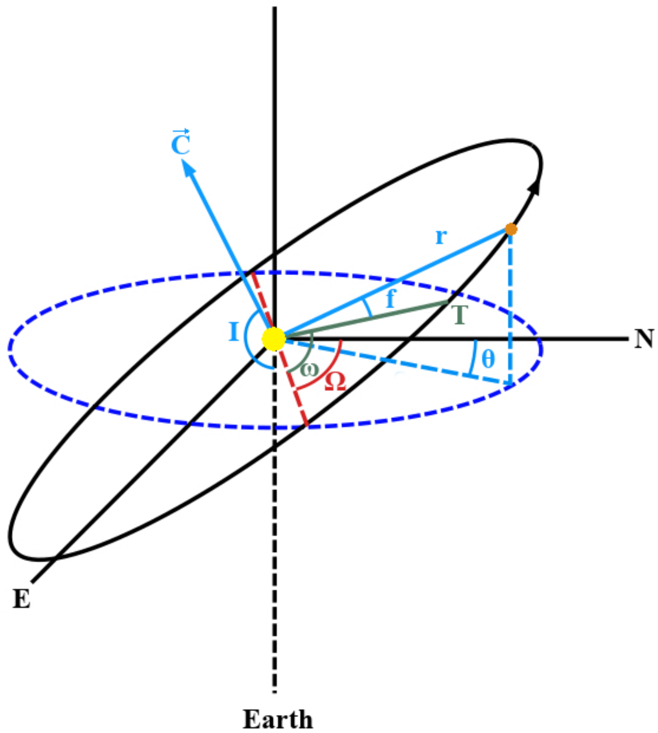

FIGURE I.5: Representation of the relative orbit (solid black line) and the aparent orbit (dashed dark blue line). We can see also represented the inclination (I), the angle of the node (Ω) and the argument of the periastron (ω), as long as the distance (r), the true anomaly (f) and the position angle (θ). The line of the nodes is indicated with a dashed red line and the periastron is in the position marked with a T.

whereas for retrograde motion, as the one represented in Figure I.5, $I > 90°$. The apparent orbit is the cylindrical projection of the relative orbit onto the reference plane. Both orbits coincide at two points which are called “nodes”.

The ascending node is the one in which the star moves away from the Sun. Ω is measured on the reference plane with respect to an origin (in this case, the North direction was chosen, following the N-E-S-W convention). The argument of the periastron is the angle that the periastron and the ascending node form on the orbit and it follows the direction of movement. The first two orbital elements are dynamical, whereas the fourth and fifth indicate the shape of the orbit, and the last three show the orientation of the orbit in space. When we study double stars, we are not able to measure the linear distance between them but we can measure their angular separation, so when we calculate the semimajor axis of the orbit, it will be given in angular units, usually arcseconds ("). For the rest of this work, unless we note it with an upper bar ($\bar{a}$), the semimajor axis will be considered as an angular quantity, measured in arcseconds. If the distance is known from the parallax, π'', we will be able to obtain the semimajor axis in a. u. from the equation:

$$\pi'' \approx \frac{1}{d} \approx \frac{a''}{\bar{a}} \tag{I.9}$$

Here π'' represents the parallax, measured in arcseconds, d stands for the distance in parsecs (pc), and a'' and $\bar{a}$ indicate the semimajor axis measured in arcseconds and a. u., respectively.

There are many methods to obtain the relative orbit of a visual double star from observations (Abad, Docobo, and Elipe, 2017). Some of them, like the methods of Zwiers or Kowalsky, assume the knowledge of the apparent orbit and they use it to derive the relative orbit by means of geometric (Zwiers, 1896) and analythical relationships (Vidal, 1957), respectively. Other algorithms compute the seven orbital elements of the relative orbit directly from seven observational data. The Thiele-Innes-van den Bos method uses three observations or three normal points $(\theta_i,\rho_i;t_i)$, $i = 1, 2, 3$, as well as the constant of the areas of the apparent orbit which is the areal velocity (constant, because we are dealing with a Keplerian motion) $c = \frac{\pi a^2\sqrt{1-e^2}}{P} cos\ I$. This value needs to be determined previously by using all of the available observations. The method of Cid (Cid Palacios, 1958) requires three complete observations $(\theta_i,\rho_i;t_i)$, $i = 1, 2, 3$, and another incomplete measurement $(\theta_4;t_4)$ to obtain the seven orbital elements with the necessary and sufficient number of observational data. Dopico (Dopico, 1961) solved the same problem geometrically. Previous to that, E. Vidal (Vidal, 1953; Vidal, 1948) had solved the parabolic case with three points $(\theta_i,\rho_i;t_i)$, $i = 1, 2, 3$. In 1979, Monet designed (Monet, 1979) a method that used Fourier transforms which was an adaptation of the Wilsing-Russell algorithm that was used for spectroscopic binaries (Russell, 1902). Nowadays, thanks to the computing power of modern computers, Monte Carlo methods are widely used, particularly the Markov Chain Monte Carlo methods (MCMC), such as the one designed by R. Méndez et al. (Mendez et al., 2017).

The method that is used at OARMA was developed by J. A. Docobo (Docobo, 1985, 2012; Docobo, Tamazian, and Campo, 2018). It consists of an analithycal algorithm that uses only three base points $(\theta_i,\rho_i;t_i)$, $i = 1, 2, 3$. In principle, this data is not enough to determine the orbit but they generate a family of orbits that pass through these three points (and which can be empty). An orbit is selected from the family following different criteria, the most ususal is the minimum root-mean-square error (rms) determined from the differences between the observations and the calculated values of θ and ρ for the positions in those epochs (ephemerides). Other criteria are the masses obtained for the components of the system and their physical plausability, the difference between the parallax calculated from the orbital elements and the spectral types (dynamical parallax) and an observed value (by the satellites *Hipparcos* or *Gaia*, for example), the adjustment between the common orbital elements obtained by means of other techniques, such as radial velocities, etc. Besides, as for every orbit of the family, we can determine the constant of the areas. It is enough to compare those results with the value of that constant calculated from all the observations. Using the constant of the areas as a control, it would be like working with the Thiele-Innes-van den Bos method.

The basis of this method consists of establishing a mapping from the interval (0,2π) into the set of elliptical Keplerian orbits for which their apparent orbits pass through the base points. Obviously, this base points are chosen to match high quality observations.

If E_3 and E_1 are the eccentric anomalies corresponding to the epochs, t_3 and t_1, respectively, an orbit is obtained for each value $V = E_3 - E_1$, although there may be values of V without a periodic solution (Docobo, 1985, 2012).

In this way, if the three points are separated by less than a period, we have that $V \in (0, 2\pi)$, unlike with the constant of the areas that, when $V \longrightarrow 2\pi$, it turns out that $c \longrightarrow +\infty$. Anyway, c is univocally determined by V as long as the three points are in the same revolution. Also, each orbit obtained for a value of V will yield different ephemerides out of the three base points. In essence, the methods of Thiele-Innes-van den Bos and Cid are particular cases of the Docobo Method.

It is a very versatile method that has been used to calculate hundreds of double star orbits, both by the OARMA personnel and by other researchers. The author of this Memory has developed an implementation of the method using Matlab, derived from the original program in Fortran, that includes the following features:

- The Docobo Method.
- The calculation of ephemerides from the orbital elements and the evaluation of the rms error with respect to a set of observations.
- The precession correction of the observations.
- The calculation of the dynamical parallax from the orbital elements, the difference in magnitude between the components, and the spectral types by means of the Baize-Romani algorithm (Baize and Romani, 1946) with the calibrations given by Docobo and Andrade (Docobo and Andrade, 2013).
- The calculation of the sum of the masses of the components from the dynamical and trigonometric (*Hipparcos* or *Gaia*) parallaxes and the individual masses from the dynamical parallax and the difference in magnitude.
- The graphical representation of the orbit and the observations.
- The improvement of the orbit by means of a least squares minimization of the sum of the rms in θ and ρ.

- The calculation of the radial velocity ephemerides and their rms with respect to a set of observations.
- The representation of the radial velocity curves.

This implementation of the Docobo Method has been used by Professor Docobo and the author of this Memory in the articles corresponding to the calculation of binary star orbits included in this dissertation as well as other articles published in recent years and orbits published in the Information Circular of the IAU Commission G1 (Binary and Multiple Star Systems, http://www.usc.es/astro/circularing.html) with the author code, Docobo-Campo. The determination of the orbits permits the calculation of the sum of the masses of the components in units of solar mass ($\mathcal{M}_\odot$) from Kepler's Third Law, as long as we know the parallax (trigonometric or dynamical), by using the following expression:

$$\mathcal{M}_1 + \mathcal{M}_2 = \frac{\bar{a}^3(u.a.)}{P^2} = \left(\frac{a('')}{\pi('')}\right)^3 \frac{1}{P^2} \tag{I.10}$$

in which the orbital period, P, must be given in years.

I.1.3 The Baize-Romani algorithm

This algorithm which was presented by P. Baize and L Romani in 1946 (Baize and Romani, 1946; Heintz, 1978) permits the calculation of the parallax of a binary system from its orbital elements (concretely, the semimajor axis and the period), the spectral types of the components, the difference in magnitude (Δm), and the total magnitude of the system. On one hand, it is based on Kepler's Third Law and, on the other, on a mass-luminosity relationship (MLR) of the stars as follows:

$$L \propto \mathcal{M}^k, \tag{I.11}$$

for a value of k. If we define $h^3 = (\mathcal{M}_1 + \mathcal{M}_2)\pi^3$, where π is the parallax, given in arcseconds, and take it into the equation I.10, we obtain for the semimajor axis, a, in arcseconds, and the period, P, in years:

$$log(h) = log(a) - \frac{2}{3}log(P) \tag{I.12}$$

The luminosity of a star, L, can be written in terms of the absolute bolometric magnitude, $L = 10^{0.4(M_{Bol}-M_0)}$ (in units of solar luminosity, $L_\odot$), with M_{Bol} the bolometric magnitude of the star, and M_0 the reference bolometric magnitude. In the case of main-sequence stars, M_0 is usually chosen as the bolometric magnitude of the Sun. In order to obtain the bolometric magnitude, we need to measure the luminosity along the entire electromagnetic spectrum but it can be calculated from the visual magnitude by means of the equation, $M_{Bol} = M_V + BC$, where BC is called the bolometric correction, and it is determined empirically. There are multiple calibrations that list these corrections, for example, those included in Bessell, Castelli, and Plez (1998), Flower (1996), Gray (2005), and Straizys and Kuriliene (1981). In principle, any calibration can be used as long as we are careful to correct them according to the zero-point (Torres, 2010). We can reformulate the equation I.11 in terms of the bolometric magnitudes:

$$M_{Bol} = M_0 - \frac{5}{2}\ k\ log(\mathcal{M}) \tag{I.13}$$

And taking into account the known relationship between the absolute magnitude, M, and the apparent magnitude, m:

$$M = m + 5 + 5log(\pi), \tag{I.14}$$

which is valid both for the visual and bolometric magnitudes and, if we define:

$$\begin{aligned}
A = 2.5[&log(1 + 10^{-0.4\Delta m + 0.4(BC_2 - BC_1)}) && \text{(I.15)}\\
&- k\ log(1 + 10^{(-0.4\Delta m + 0.4(BC_2 - BC_1))/k}) && \text{(I.16)}\\
&- log((1 + 10^{-0.4\Delta m + 0.4(BC_2 - BC_1)})/(1 + 10^{0.4\ \Delta m}))] && \text{(I.17)}\\
B = &\ m_1 - 2.5\ log(1 + 10^{0.4\Delta m}) && \text{(I.18)}\\
C = &\ BC_1 + 5 - M_0 && \text{(I.19)}\\
D = &\ 7.5\ k\ log(h) && \text{(I.20)}
\end{aligned}$$

we can combine the equations I.12, I.13 and I.14 in order to obtain the dynamical parallax:

$$log(\pi_{dyn}) = \frac{A + B + C + D}{7.5k - 5}. \tag{I.21}$$

This parallax is quite accurate for main-sequence stars because their MLR is well determined, and it is reasonably accurate for subgiant stars whereas, for giant and supergiant stars, it is not applicable. With this parallax, we can calculate the sum of the masses of the components using Kepler's Third Law, as well as the individual masses by means of the relationship:

$$log(\mathcal{M}_1 + \mathcal{M}_2) = 3log(h) - 3log(\pi) \tag{I.22}$$

$$k\ log(\frac{\mathcal{M}_1}{\mathcal{M}_2}) = 0.4\Delta M \tag{I.23}$$

The algorithm is based on two parameters, k and M_0, which are obtained empirically, and they depend on the chosen calibration. In our implementation of this algorithm that we use with Docobo's method, we take the values provided by J. A. Docobo and M. Andrade (Docobo and Andrade, 2013) that are based on the calibrations of V. Straizys and G. Kuriliene (Straizys and Kuriliene, 1981). The adopted values are $k = 4.23$ and $M_0 = M_{B\odot} = 4.74$ for main-sequence stars and $k = 3.64$ and $M_0 = 3.88$ for subgiants.

I.1.4 The Edwards process

As we have seen in the previous section, the knowledge of the individual spectra permits us to obtain information about the astrophysical parameters of the components of the system. In the case of single stars, the spectrum can be measured directly by using a spectrometer but, when we have a binary system, if the separation between the components is lower than the resolution of the spectrograph, we will not be able to get both spectra. We will obtain a combined spectrum in which features of the spectra of both components appear unless the difference in magnitude between the components is high.

There are several methods to separate the spectra of the binary components. If we can determine, or at least estimate, the spectrum of one of the components (for example, during a total eclipse in the case of eclipsing binaries), we can substract it from the combined to obtain the one from the other star (Griffin and Griffin, 1986). In other cases, the spectral features of each component can be easily identified in the combined spectrum, and the process of separation is essentially straightforward (Ferluga et al., 1997; Pilachowski and Sowell, 1992). However, if the spectral lines of the components overlap, this is not possible and we need to resort to more sophisticated techniques. One option is the use of tomographic algorithms for image reconstruction (Bagnuolo and Gies, 1991). Also, K. P. Simon and E. Sturm (Simon and Sturm, 1994) developed a method based on obtaining several combined spectra from different phases (non-eclipsing) that are later transformed in an over-determined, rank-deficient system which can be solved. This method also permits the determination of the elements of the spectroscopic orbit. Finally, P. Hadrava designed a method based on the Fourier transform to determine both the spectra and the spectroscopic orbit (Hadrava, 1995).

This procedures are widely used in the study of spectroscopic and eclipsing binaries. Many interferometric visual binaries are also spectroscopic or eclipsing because they are close systems, therefore their spectra are determined. However, for wider systems with lower radial velocities, the spectra overlap completely and even the most sophisticated techniques are unable to separate them. Thus, we only know the combined spectrum in many visual systems that have not been observed with high resolution.

In 1976, T. W. Edwards (Edwards, 1976) proposed a method to obtain the individual spectra of binaries from the combined and the difference in magnitude by using a calibration of the luminosities of the MK system. The article was based on a previous work by J. W. Christy and R. L. Walker (Christy and Walker, 1969) and it consists of performing an interpolation of the spectral types by means of the following expressions:

$$S(1) + xS(2) = (1 + x)S(1 + 2), \ and \tag{I.24}$$

$$M[S(1)] - M[S(2)] = \Delta m = -2.5log(x). \tag{I.25}$$

where $S(1)$, $S(2)$, and $S(1+2)$ represent the spectral types of the primary, the secondary, and the combined, respectively. $M[S]$ is the calibration of the luminosity and Δm is the difference in magnitude. However, there is the question of how to interpret the quantity that represents the spectral type, $S(i)$, in order to be able to interpolate it. A first approximation is to use a calibration of the absolute magnitude, M_V, of the combined spectrum and to obtain the individual magnitudes from the equation:

$$M_{V1} + x\, M_{V2} = (1 + x)M_V, \tag{I.26}$$

in which M_{Vi} stands for the absolute visual magnitude of the component i and x is calculated from Δm and the second equation of I.24. Although this approximation yields satisfactory results when it is applied to real systems, its physical interpretation is more problematic. Another approximation is based on the work by W. I. Beavers and D. B. Cook (Beavers and Cook, 1980). In order to follow this work, it is necessary to clarify several concepts beforehand.

The thermal radiation produced by a star is similar to that of a black body with the same surface temperature. The brightness per surface unit of a black body in thermodynamical equilibrium at temperature T, for a wavelength, λ, can be calculated from Planck's Law:

$$B_\lambda(T) = \frac{2hc}{\lambda^5} \frac{n_\lambda^2}{exp(\frac{-hc}{\lambda kT}) - 1}, \tag{I.27}$$

where h and k are the Planck and Boltzmann constants, respectively, n_λ represents the refraction index of the medium for that wavelength, and c is the speed of light in the free space. If we integrate the brightness with respect to the wavelength, we obtain the Stefan-Boltzmann Law which, applied to a semi-sphere yield the luminosity, L, of a star, i.e., the total emission of energy per unit of time:

$$L = 4\pi\sigma R^2 T^4. \tag{I.28}$$

R represents the radius of the star, T stands for the surface temperature, and σ is the Stefan-Boltzmann constant. For a certain wavelength, λ, the spectral flux density, f_λ, is defined as the amount of energy incoming from a star (with R radius and T temperature) that crosses a unit of surface at a distance (D) and it can be calculated from the brightness by means of the following expression:

$$f_\lambda(T) = \frac{\pi R^2}{D^2} B_\lambda(T). \tag{I.29}$$

If we integrate f_λ with respect to the wavelength, it yields the flux, which is related to the luminosity and the distance through the equation:

$$F = \frac{\pi R^2 L}{D^2}. \tag{I.30}$$

There are also relationships between both the spectral flux density in the visual band and the visual magnitude, and between the flux and the bolometric magnitude:

$$m_V = -2.5 log(f_V), \tag{I.31}$$

$$m_{Bol} = -2.5 log(F). \tag{I.32}$$

Finally, the normalized flux for a given wavelength λ is defined as:

$$F_\lambda = \frac{f_\lambda}{F}. \tag{I.33}$$

Back to the work of Beavers and Cook, the basis of it is that, given a binary system and a constant $k \geq 0$, the monochromatic normalized flux of the system, F_λ, can be obtained from the individual fluxes ($F_{1,\lambda}$, $F_{2,\lambda}$) as follows:

$$F_\lambda = \frac{F_{1,\lambda} + k\ F_{2,\lambda}}{1 + k}. \tag{I.34}$$

The similarity with the Edwards formulation is evident and if we consider $k = \frac{F_2}{F_1}$, the equation I.34 can be written in the following way:

$$\frac{f_\lambda}{F} = \frac{\frac{f_{1,\lambda}}{F_1} + k\frac{f_{2,\lambda}}{F_2}}{1+k}. \tag{I.35}$$

which is more physically sound than the equation I.26 but it requires the knowledge of the flux. However, if we combine the I.35 and I.31 equations, we obtain:

$$(1+k)\frac{10^{-0.4m_V}}{10^{-0.4m_{Bol}}} = \frac{10^{-0.4m_{1,V}}}{10^{-0.4m_{1,Bol}}} + k\frac{10^{-0.4m_{2,V}}}{10^{-0.4m_{2,Bol}}}. \tag{I.36}$$

and if we consider the relationship between the apparent and absolute magnitudes given in the equation I.14, it yields:

$$(1+k)\frac{10^{-0.4M_V}}{10^{-0.4M_{Bol}}} = \frac{10^{-0.4M_{1,V}}}{10^{-0.4M_{1,Bol}}} + k\frac{10^{-0.4M_{2,V}}}{10^{-0.4M_{2,Bol}}}. \tag{I.37}$$

Finally, if we take into account the relationship between the visual and bolometric magnitudes, $BC := M_{Bol} - M_V$, the equation I.37 is equivalent to:

$$(1+k)\ 10^{0.4\ BC} = 10^{0.4\ BC_1} + k\ 10^{0.4\ BC_2}. \tag{I.38}$$

The bolometric correction of the combined spectrum (BC) can be obtained from any calibration and now the equation I.24 of the Edwards method is shown in terms of the bolometric correction, taking into account that $k = x = 10^{-0.4\ \Delta m}$. In order to determine the spectral types, we will consider the bolometric correction as a function of the absolute magnitude, $BC = \phi(M_V)$, that can be interpolated from the calibration values and the order of the equation I.38 can be reduced by means of the difference in magnitude:

$$(1+k)10^{0.4\ BC} = 10^{0.4\ \phi(M_{1,V})} + k\ 10^{0.4\ \phi(M_{1,V}+\Delta m)} \tag{I.39}$$

and it can be solved numerically to obtain $M_{1,V}$ and, therefore, $M_{2,V}$. The spectral types are obtained from the calibration.

I.1.5 Application of the methodology

We will now apply the methodology to several visual double stars with calculated orbits. In these examples, we have included only main-sequence stars although it could be applied to subgiant stars taking into account the indications by Edwards about the assignment of luminosity classes. The absolute magnitude of giant stars is not monotonically increasing and, therefore, it cannot be used for the interpolation of the bolometric correction. We have selected the calibration of Straizys and Kuriliene (1981) and we have taken the spectral types provided by the SIMBAD database, operated by the CDS in Strasbourg, France. In order to determine the differences in magnitude, we have used the values obtained by means of the speckle measurements included in the Fourth Catalog of Interferometric Measurements of Binary Stars of the U.S. Naval Observatory. When there are several values of Δm, we calculate the arithmetic mean of those close to the V band. The results are depicted in Table I.1. In the first two columns the WDS designation and the name of the system are included, respectively, whereas the third column shows the difference in magnitude. The combined and the individual spectral types are represented in columns 4, 5, and 6. If two solutions are possible, correponding to two different measurements of the combined spectrum, they appear in consecutive rows.

The interpolation was performed with the method of cubic splines, by using the "interpolate" package included in the "scipy" library of Python. The equation I.39 was solved by means of a Newton method.

In the Table I.2, we compare the values obtained in the original work by Edwards with those calculated with our implementation. Therefore, we use the values of the spectral types and differences in magnitude included in the article by Edwards. Part of the slight differences between both works may be attributed to the use of different calibrations. The first four columns follow the format of the previous Table and the last four columns show the individual spectral types, first those calculated by Edwards and, finally, ours.

I.2 Spectroscopic binaries

Spectroscopic binaries are detected thanks to the Doppler shift that occurs in the spectral lines of the components throughout their periodic movement toward and away from the Earth. This shift can be used to calculate the radial component of the orbital velocity of the binaries by means of the following equation:

$$\frac{v_r}{c} = \frac{\lambda_{obs} - \lambda_0}{\lambda_0} \tag{I.40}$$

where v_r represent the radial velocity; c, the speed of light in free space; λ_{obs}, the observed wavelength of a determined spectral line; and λ_0, the wavelength of that spectral line at rest, as measured in the laboratory. This relationship is valid as long as the observer and the source of radiation are point objects moving with repect to each other which, in reality, will not be true and that is why we need to include several terms of correction. First, we have a daily term

TABLE I.1: Examples of the application of the methodology

WDS	Name	Δm	Sp. type (combined)	Sp. type (A)	Sp. type (B)
01477 − 4358	I52	0.62	F6/7V	F5V	F8V
				F5V	F9V
02514 − 2139	DON43	0.2	F3V	F2V	F4V
03189 − 0101	BU1177	0.18	G8V	G7.5V	G8.5V
04142 − 4608	RST2338	1.34	F8/G0V	F5V	G3V
				F8V	G7V
04506 + 1505	CHR20	0.8	G5	G2.5V	G8V
06274 − 2544	B114	0.28	K0V	G9V	K1V
07013 − 0906	A671	0.6	F5	F4V	F8V
12155 − 3106	RST1658	1.15	K7Vk	K6V	M0V
13044 − 1316	HU642	0.31	G0	F9V	G1V
14243 − 3838	RST1785	0.27	G5V	G4V	G6V
15332 − 2429	CHR232	1.76	A7V	A6V	F6V
16094 − 3103	I557	0.8	A7IV	A5IV	F0IV
				A6IV	A7V
17115 − 1630	HU169	0.695	A7V	A6V	F0V
18434 − 5546	B398	1.23	F7V	F4V	G2V
19264 + 4928	YSC134	0.95	K2.5V	K1.5V	K5V
20081 − 3929	RST2134	0.42	G0V	F9V	G2V
22007 − 5002	I1450	0.19	K0V	K0V	K0.5V

TABLE I.2: Comparison between the Edwards results and this work

WDS	Name	Δm	Sp. type (combined)	Edwards Sp. type (A)	Edwards Sp. type (B)	This work Sp. type (A)	This work Sp. type (B)
00318 + 5431	STT12	0.18	B8V	B7.5V	B8.5V	B7.5V	B8.5V
02396 − 1152	FIN312	0.15	F6V	F6V	F6V	F6V	F6.5V
03175 + 6540	STT52	0.45	A3V	A2V	A4V	A2.5V	A5V
04199 + 1631	STT79	1.38	F9V	F7V	G6V	F6V	G5V
05017 + 2640	A1844	1.39	G2V	G0V	G8V	G0V	G9V
06474 + 1812	STT156	0.22	A2V	A1V	A3V	A1.5V	A2.5V
07175 − 4659	I7	0.70	K2V	K1V	K4V	K1V	K4V
08394 − 3636	I314	1.45	F3IV	F2IV	F6V	F2IV	F7V
09210 + 3811	STF1338	0.25	F3V	F2V	F4V	F2V	F4V
10361 − 2641	BU411	0.98	F6V	F4V	G0V	F4V	G0V
11047 − 0413BC	A676	0.10	M0V	M0V	M0V	M0V	M0V
12396 − 3717	DAW63	0.35	K5V	K4V	K6V	K4.5V	K6V
13123 − 5955	SEE170	0.41	B8V	B8V	B9V	B7.5	B9V
14463 + 0939	STF1879	0.62	G2V	G1V	G4V	G1V	G5V
17082 − 0105	A1145	2.00	A3V	A1V	F3V	A3V	F4V
18570 + 3254	BU648	2.20	G0V	F9V	K1V	F8V	K1V
20474 + 3629	STT413	1.26	B5V	B4V	B7V	B4V	B7.5V

with a small value caused by the rotation of the Earth. In addition to this, the radial velocities are referred to the baricenter of the Solar System, therefore, we need to include a small correction term due to the movement with respect to the baricenter of the Earth-Moon system and a larger correction due to the movement with respect to the Earth-Sun baricenter which, in the case of binary star research cannot be neglected. It is also nescessary to determine the radial velocity of the center of mass of the binary system with respect to the baricenter of the Solar System, γ which, contrary to the other terms, is not known and is considered one of the spectroscopic orbital elements to be obtained. Besides, in very close stars, the relativistic effects can not be neglected and we have to include them. Finally, there may be other contributions to the radial velocity signal, for example, systematic effects due to the detector or the presence of other bodies in the system. The radial velocity of a star, when measured through time, will describe a periodic curve called the radial velocity curve, the shape and dimensions of which will depend on the orbital parameters.

I.2.1 Observation

The observable quantities for this type of binaries are the radial velocities of the components. There are absorption lines for each of them that will shift in opposite directions because, in the orbit of the system, when one of the components moves towards the Earth, the other moves away from it and viceversa. When we can observe the lines of both components, they are called double-lined spectroscopic binaries (SB2). Figure I.6 shows the light curves of a binary of this type. However, this is not always the case because, if the magnitude difference is high (usually more than 1.5 or 2), the light of the primary will cover the spectral lines of the secondary and they will not be visible. It is also possible, in cases in which the radial velocitites are small, that the spectral lines of the components overlap and the velocity of the secondary cannot be determined. In any case, when we can only obtain the radial velocity of the primary, the system is called a single-lined spectroscopic binary (SB1). An example of an SB1 light curve is shown in Figure I.7. Obviously, if the orbit of the system is perpendicular (or nearly so) to our line of sight, the radial component of their velocities will be zero (or very small) and the system can not be studied with this technique.

The receivers that permit to measure the spectrum of a star are called spectrometers. They usually don not measure the whole spectrum, but they are restricted to a bandwidth around a determined wavelength. The spectrometers that work in the visible band are called spectrogrphs. In these devices the light passes through a lens or collimator to a prism or a diffraction grid, which separates the light in the different wavelengths, and then goes through a slit, that restricts the light to a bandwidth, into a detector that performs the measurement. There are plenty of designs of spectrographs, although the most common is the one in which the intensity of light is measured in the whole bandwith. It has to be noticed that the intensity is not constant along the band, but in the case of a star it will follow with high fidelity the spectrum of a blackbody given in the equation I.27, with steep reductions in the light intensity

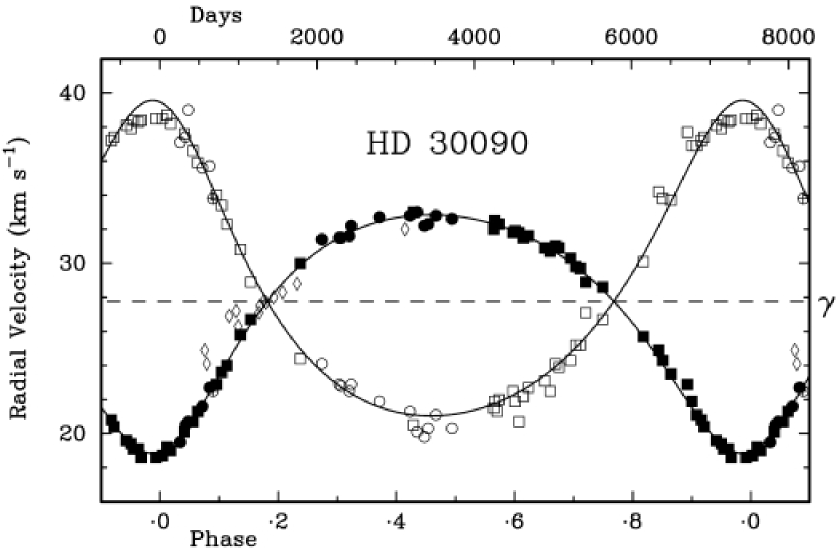

FIGURE I.6: Radial velocity curve of the double-lined binary HD30090. Included in the article (Docobo et al., 2014b).

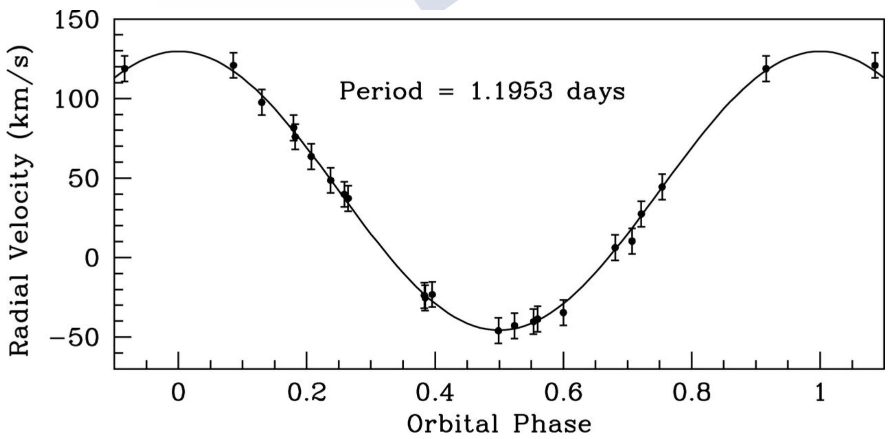

FIGURE I.7: Radial velocity curve of the single-lined binary HD99842. Included in the article (Boffin et al., 2012).

for narrow bands that match the spectral lines. Once the individual spectra are separated (if possible), they are compared by means of a cross-correlation to a synthetic spectrum that would correspond to a star at rest in order to obtain the radial velocity.

Given two continuous functions $f, g : \mathbb{R} \longrightarrow \mathbb{C}$, their cross-correlation is defined as follows:

$$(f \star g)(\tau) = \int_{-\infty}^{+\infty} \overline{f(t)} g(t + \tau) dt \tag{I.41}$$

where $\overline{f(t)}$ represent the conjugate complex. In this case the functions f and g would correspond to the intensities of the observed and the synthetic spectra, therefore the rank would be in $\mathbb{R}$, and the variable of integration would be the wavelength. We have also to take into account that in an observation the detector does not measure the continuum spectrum, but takes a sample at a rate given by the Nyquist-Shannon theorem (Nyquist, 1928; Shannon, 1949), and we need to use the dicrete version of the cross-correlation:

$$(f \star g)(n) = \sum_{m=-\infty}^{+\infty} \overline{f(m)} g(n + m) \tag{I.42}$$

I.2.2 The Cambridge Observatory

As part of the formation related with the production of this PhD dissertation and within the framework of the AYA2011-26429 research project that was developed at OARMA, the author of this dissertation worked during a research visit in 2014 at the Observatory of the Institute of Astronomy of the University of Cambridge where Professor R. F. Griffin conducts his research. He is one of the most recognized experts in spectroscopic binaries and the goal was to learn how to work with this type of system.

Professor Griffin has at his practically exclusive disposal the 0.91 m. telescope of the Cambridge Observatory (see Figure I.8) where he has installed a spectrometer of his own design in the Coudé focus. This detector is a bit different than the one that we saw in the previous section because, after the slit, it incorporates a mask with the negative of the spectrum of the star α-Bootis (Arcturus). Due to this mask, the spectrometer is transparent in the wavelengths of the spectral lines and opaque in the rest of the spectrum and, behind the mask, there is a photometer. The mask shifts sideways and when the spectral lines of the star and the mask coincide, the light is blocked which can be measured in the photometer. With this procedure, we have a measurement of the photon count and we can observe one (SB1) or two (SB2) drops in the detected intensity when the mask passes in front of the spectral lines of the components (see Figure I.9). The radial velocity can be obtained directly by measuring the separation of the mask with respect to the initial point. This design has the advantage of more rapid calculation of the radial velocities because it is not necessary to

FIGURE I.8: 0.91 m. telescope of the Cambridge Observatory. The spectrometer is located inside the black box in the red structure placed at the right side of the telescope.

perform the cross-correlation with the reference spectrum. However, it has the disadvantage that the spectrum of the star must be reasonably similar to the one in the mask and that is why he is restricted to the observation of stars of the F, G, and K spectral types. Once the measurement is made, it is usually necessary to make a small correction due to the position of the spectrometer with respect to the telescope in order to obtain the radial velocity.

Once enough measurements of radial velocities of a system have been obtained, the spectroscopic orbit can be calculated. Professor Griffin does this by means of a least squares minimization performed with a program in Fortran that he kindly copied for the author of this dissertation. Using these techniques, he has published more than 260 article of the series *Spectroscopic binary orbits from photoelectric radial velocities*, mainly in the journal *The Observatory* as well as other work in collaboration with different research groups, including ours (Docobo et al., 2014b, 2017a).

I.2.3 Spectroscopic orbit calculation

Some of the orbital elements can be deduced from the radial velocities but not all of them. We can directly obtain the period, P, the epoch of the periastron passage, T, the eccentricity, e, and the argument of the periastron, ω. In the case of this last element, we have to take into account that, for the SB1, we calculate the argument of the orbit of the main component with respect to the center of mass, ω_1, not the relative orbit. However, this is not a problem because, for a Keplerian orbit, it verifies that $\omega = \omega_1 + 180°$.

As mentioned previously, in addition to these elements we can obtain the velocity of the center of mass of the system, γ (sometimes it is also written as V_0). The last orbital parameter is the semi-amplitude of the radial velocity curve of the primary, K_1 and, in the case of SB2s,

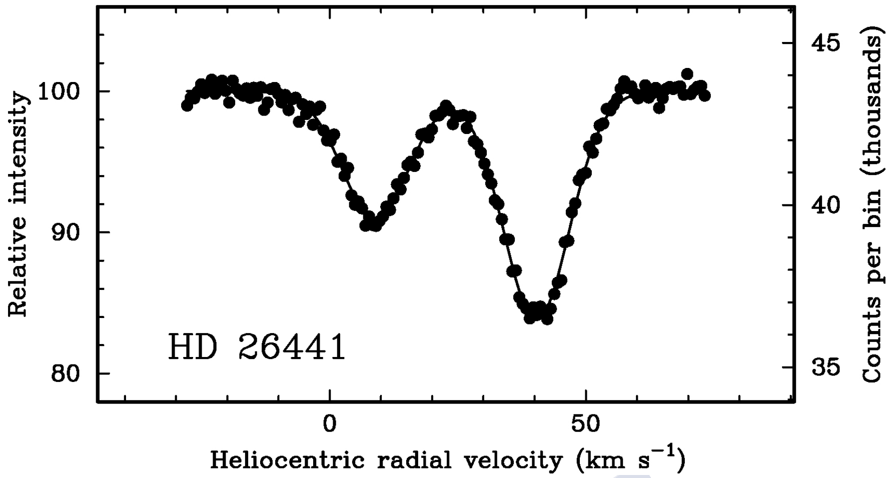

FIGURE I.9: Observation of the double-lined binary HD26441 performed in Cambridge. Included in the article (Docobo et al., 2017a).

that corresponding to the secondary, K_2. The semimajor axis, $\bar{a}$, and the inclination, I, are implicitly incorporated in these parameters by means of the following relationships:

$$K_1 = \frac{n\,\bar{a}_1 sin\ I}{\sqrt{1-e^2}}, \tag{I.43}$$
$$K_2 = \frac{n\,\bar{a}_2 sin\ I}{\sqrt{1-e^2}}, \tag{I.44}$$
$$K = \frac{n\,\bar{a}\ sin\ I}{\sqrt{1-e^2}}, \tag{I.45}$$

where $n = \frac{2\pi}{P}$ is the mean motion, and $\bar{a}_1$ and $\bar{a}_2$ are the semimajor axes of the primary and the secondary orbits with respect to the center of mass, respectively (given in units of distance). Obviously, they verify that $\bar{a}_1 + \bar{a}_2 = \bar{a}$. If we take into account that, for the two-body problem, we have the following equations:

$$n^2\bar{a}_1^3 = \frac{\mathcal{G}\mathcal{M}_2^3}{(\mathcal{M}_1+\mathcal{M}_2)^2}, \tag{I.46}$$
$$n^2\bar{a}_2^3 = \frac{\mathcal{G}\mathcal{M}_1^3}{(\mathcal{M}_1+\mathcal{M}_2)^2}, \tag{I.47}$$

If we choose the right units, $\mathcal{G} = 1$, and by combining equations I.43, I.44, I.46 and I.47:

$$K_1^2 = \frac{\mathcal{M}_2^3 sin^3 I}{\bar{a}_1 sin\ I(1-e^2)(\mathcal{M}_1+\mathcal{M}_2)^2}, \tag{I.48}$$

$$K_2^2 = \frac{\mathcal{M}_1^3 sin^3 I}{\bar{a}_2 sin\ I(1-e^2)(\mathcal{M}_1+\mathcal{M}_2)^2} \tag{I.49}$$

In the case of the SB1s, we define the mass function as:

$$f(\mathcal{M}) = \frac{\mathcal{M}_2^3 sin^3 I}{(\mathcal{M}_1+\mathcal{M}_2)^2} = K_1^2 \bar{a}_1 sin\ I(1-e^2). \tag{I.50}$$

If the binary is a SB2, we have an equivalent expression for the secondary (just exchanging the subscripts 1 and 2) and by dividing both mass functions, we obtain the mass ratio:

$$q = \frac{\mathcal{M}_1}{\mathcal{M}_2} \tag{I.51}$$

The radial velocities can be calculated from the orbital elements of the spectroscopic orbit by means of the following equations:

$$v_1 = \gamma + K_1(cos(\omega+f) + e\ cos(\omega), \tag{I.52}$$

$$v_2 = \gamma + K_2(cos(\omega+f) + e\ cos(\omega) \tag{I.53}$$

There are many methods to calculate the orbital elements of the spectroscopic orbits, both analytical and numerical. The most classical among them is the method of Lehmann-Filhés (Lehmann-Filhés, 1894; Smart and Green, 1977) which is based, on one hand, on an estimation of the period, P, and on the other on obtaining an adjustment of the radial velocity curve, for example, by means of an interpolation of the observed radial velocities. With this curve and the period, we can calculate the rest of the orbital elements. Another method is that of Wilsing-Russell (Russell, 1902; Wilsing, 1893) for which we assume a known period as well and that uses a decomposition in the Fourier series of the Equation I.52. There are also the methods of Simon and Sturm (Simon and Sturm, 1994) and Hadrava (Hadrava, 1995) that permit us to obtain the spectral types as well as numerical methods like the one used by Griffin, among many others.

I.2.4 Spectro-interferometric binaries

In the case that a system has a visual and a spectroscopic orbit, we can obtain more information about the astrophysical parameter of the system. Among the other elements, we know a'' and I from the visual orbit and, if the spectroscopic orbit is double-lined, we have $\bar{a}_1 sin\ I$ and $\bar{a}_2 sin\ I$, and we can calculate the semimajor axis in units of distance, $\bar{a} = \bar{a}_1 + \bar{a}_2$. Therefore, we can determine the orbital parallax because:

$$\pi_{orb} = \frac{a''}{\bar{a}} \tag{I.54}$$

In this way, we have a method based solely on observational data to check the parallaxes obtained by the astrometric satellites *Hipparcos* and *Gaia*. Moreover, the visual orbit yields the sum of the masses (Equation I.10) and the spectroscopic orbit provides the ratio of the masses (Equation I.51), therefore we can also calculate the individual masses of the components.

Even in the case that the spectroscopic orbit is single-lined, if we know the parallax (for example, from *Hipparcos* or *Gaia*), the mass function given in Equation I.50, as long as the inclination and the sum of the masses permit us to obtain the individual masses (Docobo et al., 2018b, see for example).

I.3 Eclipsing binaries

In this type of binaries our line of sight is contained in (or very close to) the orbital plane. Due to this fact, periodic eclipses (total or partial) are observed from Earth when their components pass in front of each other. These eclipses can be accurately studied and we can obtain some of the orbital elements, concretely the period and the inclination, along with other fundamental parameters such as the radii and the luminosities. As the information about these systems is obtained mainly during the eclipses, they are usually close systems with short periods. In this section, we will mainly follow the formulation given in Abad, Docobo, and Elipe (2017) and Kallrath and Milone (2009).

I.3.1 Observation

The eclipses cause a variation in the light that we receive from the system and the instrument that is used for their study is a photometer which measures the light intensity from a source. When we observe these stars along time, we obtain the so-called light curve (see Figure I.10) which has an approximately constant value except during the eclipses when a drop in

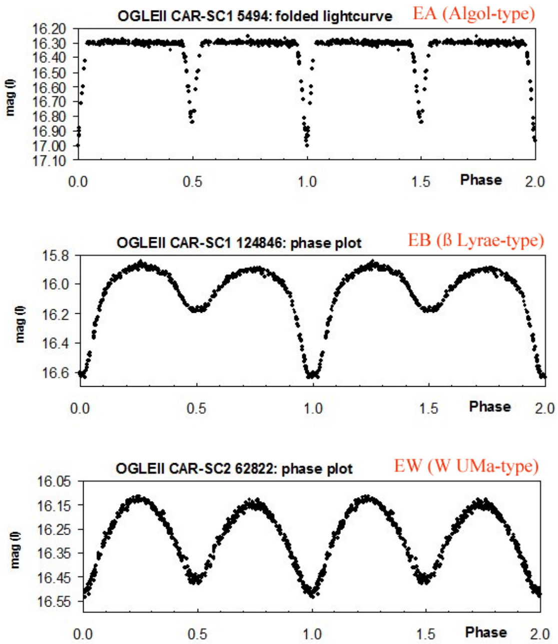

FIGURE I.10: Light curves of eclipsing binaries of the types EA, EB and EW. Image: Included in the article (Hümmerich, Bernhard, and Srdoc, 2013).

the intensity of light is detected. Unless there are perturbations in the system, the eclipses occur at regular intervals, therefore, these variations are periodic and there are two sets of different eclipses unless both stars have the same size and luminosity. When the brightest star is behind the other, we have the main eclipse with a deeper drop in the luminosity and, when the faintest star is covered, the secondary eclipse occurs.

However, not all eclipses are equal and we may produce a classification of these stars on the basis of the typology of their light curves. In this way, we have the following eclipsing binaries of the following types:

- EA: they are also called Algol-type or β-Persei due to the star that presents this type of curve that is best known. Within this class, the eclipses stretch through a relatively narrow part of a phase (the representation of one period in the light curve). Outside the eclipses, the light curve is almost flat which indicates little interaction between the components and the minima of the eclipses have different depths which indicates a large difference in brightness.

- EB: or β-Lyrae, they show a continuous variation of the brightness along the phase because the proximity between the components cause them to adopt an ellipsoidal shape due to the tidal distortion. The minima have different intensities.

- EW: also known as W-Ursa Majoris, their brightness changes continuously which indicates tidal distortion but the depth of their minima is similar.

Several physical effects may also affect the morphology of the light curve. First, we have to take into account the known effect of limb darkening. It consists of a higher brightness in the center of the star than at the border, therefore, the minimum of the eclipse is not flat but also varies continuously. Another known phenomenon that has to be taken into account is the reflection effect which causes the situation that half of each star that is directed to the other is heated by the irradiation received from its companion. If the largest star is eclipsed, the increase of the temperature will increase the brightness of the non-eclipsed part of the visible half of the star, thereby reducing the depth of the minimum. Other sources of perturbation may include starspots on the surfaces of the components, light from a third component, or the presence of circumstellar matter in the system. All of these effects must be considered in order to obtain an accurate model of the system.

I.3.2 Morphology

These systems are generally very close and the approximation of considering the stars as point objects, or even homogeneous spheres, may introduce large errors in the model. That is why it is necessary to consider tidal effects that cause the stars to acquire an ellipsoidal shape. The tidal forces may also circularize the orbit and induce the coplanarity of the equatorial planes and the stars may start to rotate synchronously or even to exchange mass. In many cases, the effects of the radiation pressure are not negligible and must be incorporated.

A classification of close binaries can be done according to them filling their respective Roche lobes. If we assume as a first approximation that the gravitational potential can be determined considering the stars as point masses and adding a centrifugal term, this potential is conservative and we can apply the classical study of the restricted three-body problem, obtaining the zero-velocity (or equipotential) surfaces, and the five Lagrangian points of the system are defined. The inner Lagrangian point, L_1, determines the largest equipotential surfaces that contain each component alone and they are called Roche lobes. We have four possible scenarios.

- Both stars are smaller than their Roche lobes. In this case, they are called *detached* systems and the components do not exchange mass in a significant way.

- One of the stars is bigger than its Roche lobe. These systems are known as *semi-detached* and said component loses matter which is absorbed by the companion or it remains as circumstellar matter.

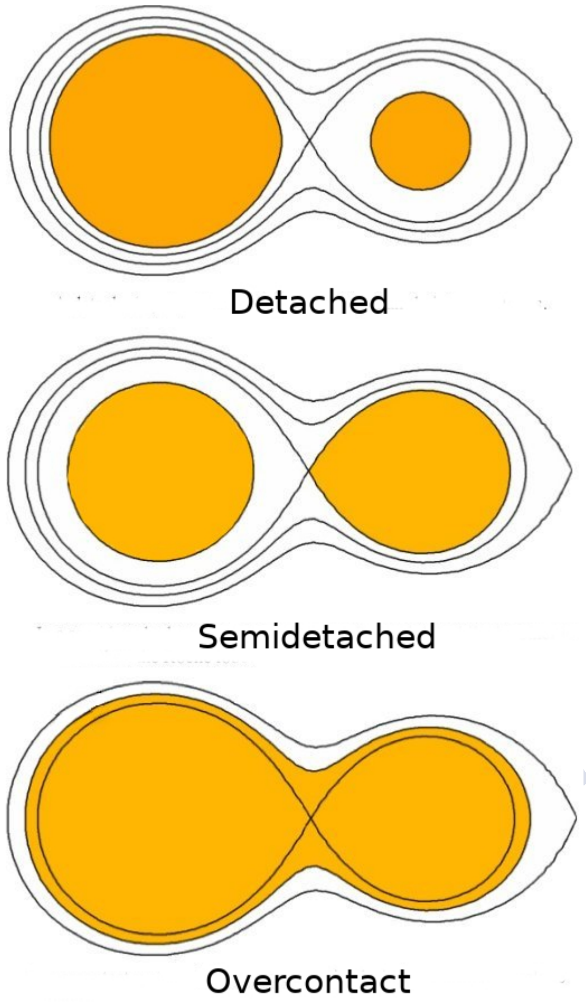

FIGURE I.11: Diagram of the Roche lobes of detached, semi-detached, and over-contact binaries. Image: Original included in the article (Terrel, 2002).

- Both stars are bigger than their Roche lobes. The components are joined by a bridge of matter that connects them,and they share a common envelope. This is the case of the *over-contact* binaries. In these systems, the stars have synchronous rotation and the orbit is circularized.

- As a particular case of the former, both stars may fill exactly their Roche lobes. They are usually systems in which one of the components absorbs matter from its companion. The star that gives mass fills its lobe and rotates synchronically, whereas the other spins faster due to the absorption of matter and its lobe shrinks due to the increase in the centrifugal force until it coincides with the surface of the star. These kinds of systems are called *double-contact* binaries and their orbits also tend to circularize.

I.3.3 Determination of the parameters

Once we have at least one light curve of the system at our disposal, we can determine some of the orbital and astrophysical parameters of the components. Concretely, we can obtain the period P, the eccentricity, e, the inclination, I, and the argument of the periastron, ω, of the orbit, as well as the ratio between the luminosities, $\frac{L_2}{L_1}$, the ratios between the radii and the semimajor axis of the orbit, $\frac{R_i}{a}$, i=1,2, their surface gravities g_i, i=1,2, and their rotational parameters. Because they are close systems and their orbital plane must be close to the line of sight, the components will have high radial velocities and the system is optimal for the study as a spectroscopic binary. If the binary is SB2, we know i and $a\ sin\ i$ and we can obtain all of the orbital elements and the individual masses, as well as their radii.

The calculation is usually performed by means of a least squares minimization, adjusting the model according to the type of binary that we are studying (EA-EB-EW, detached, semi-detached, over-contact, double-contact) and the rest of the contributions that need to be taken into account for each particular case.

It must be noticed that these parameters may vary with time. We already commented that the orbits can become circular ($e = 0$) in over-contact and double-contact binaries. Besides, mass-loss and mass-transfer phenomena also affect the period and, consequently, the semimajor axis.

In the case of mass transfer, if it occurs through the inner Lagrange point and in circular orbits when the mass goes from the least to the most massive star, the semimajor axis increases while, in the opposite case, it diminishes (Andrade, 2007; Negu and Tessema, 2015). If one of the components shows strong stellar winds, the mass transfer may happen through the accretion of those winds by the other star, although in that case, the mass absorbed is a small portion of the mass lost.

If the system loses mass, the energy and the angular moment are not conserved, and the behavior of the system will be influenced by the mass-loss mechanism and its intensity. M. Andrade in his PhD dissertation (Andrade, 2007) performed an exhaustive study of the scenarios of mass-loss in binaries taking perturbative phenomena into account, focusing on the periastron effect as well as non-spherical shapes and relativistic effects. The periastron effect causes the intensification of mass-loss when the distance between the components decreases. When we have a time-dependent mass-loss (without a periastron effect), the semimajor axis usually undergoes a secular increase and the eccentricity varies periodically. The periastron effect causes the eccentricity to increase secularly, whereas the semimajor axis and the period diminish. If we combine both types of mass-loss, the result will depend on the dominant contribution, according to the initial conditions (Andrade and Docobo, 2003). The secular influx of the non-spherical shape and the relativistic effects (that must be taken into account because we are working with close systems) can be seen in the variation of the argument of the periastron.

I.4 Multiple stellar systems

Previously, we have been considering systems with two components but there are others with more stars as is the case of α-Centauri (3), Polaris (α-Ursa Minoris, 3), Capella (α-Aurigae, 4), or Castor (α-Geminorum, 6). According to several different studies, the rate of multiplicity in our Galaxy depends on the spectral type and it is lower for the later spectral types (Duchêne and Kraus, 2013). The most massive stars (O and B types) are found in multiple systems in more than 70% of the cases, half of which contain three or more components (Peter et al., 2012; Sota et al., 2014). Around 68% of intermediate mass stars (A type) are multiples and close to 60% of them would have only two components (De Rosa et al., 2014). The stars with a mass close to that of the Sun (F and G types) belong to multiple systems inbetween 45% and 65% of the cases, with 35% of them in systems with three or more components (Duquennoy, Mayor, and Halbwachs, 1991; Fuhrmann et al., 2017; Raghavan et al., 2010; Tokovinin, 2014). The rate of multiple stars in the case of red dwarfs (M type) is around 25%, 85% of which are binaries (Ward-Duong et al., 2015; Winters et al., 2019). For brown dwarfs, the percentage is lower than 20% (Fontanive et al., 2018).

When we work with multiple systems, the orbits are not Keplerian and we would need to solve a general three-body problem. However, when we deal with hierarchical systems, we can use the theory of perturbations to analytically solve the system of equations (Abad and Docobo, 1988; Docobo, 1977; Harrington, 1969). A hierarchical system is one that can be decomposed into subgroups that are smaller each time, with only one or two components in the subgroups of the last decomposition and the distances within a level of decomposition much smaller than the distances in the upper level. The simplest hierarchical case, a triple system, consists of a double subsystem with the third component much farther away. If we increase the number of components, there are more possible cases. For example, for quadruple stars, there are two possible configurations that maintain the hierarchy, two double subsystems with a large separation between them and a double subsystem with a third component much farther away and another one at a farther distance (see Figures I.12 and I.13.

The hierarchical systems are not rare and they are probably the most common within the multiple stellar systems because they are usually more stable. In order to analytically solve the equations of the hierarchical three-body problem, the Hamiltonian of the problem is usually expressed in terms of a small constant parameter (or several parameters in the case of higher order problems) in order to eliminate the non-significant terms, therefore simplifying the equations. In hierarchical systems, the ratios of the orbital semimajor axes are commonly used as parameters although any other can be considered as long as they are small enough. A. Abad, in his PhD dissertation(Abad, 1984; Abad and Docobo, 1988), developed a methodology called the step decomposition, to systematically decompose hierarchical systems and formulate the Hamiltonian and he applied it to study hierarchical triple and quadruple (2+2 configuration) stellar systems.

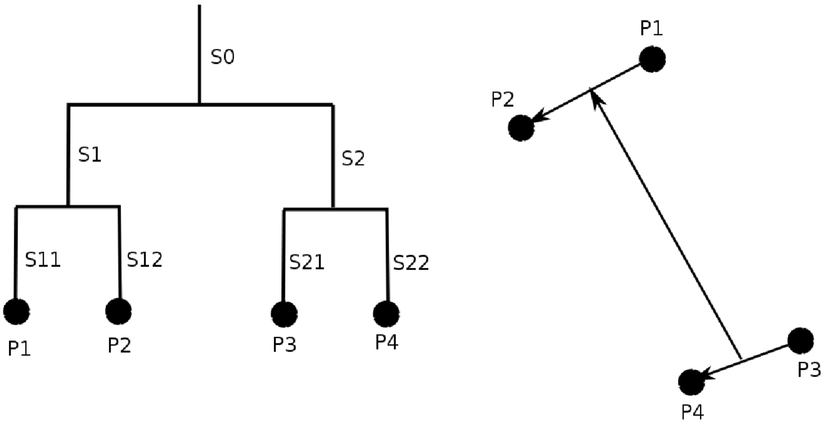

FIGURE I.12: Right hand image: hierarchical quadruple system with two double subsystems (P_1+P_2 and P_3+P_4). Left hand image: schematics of the hierarchy.

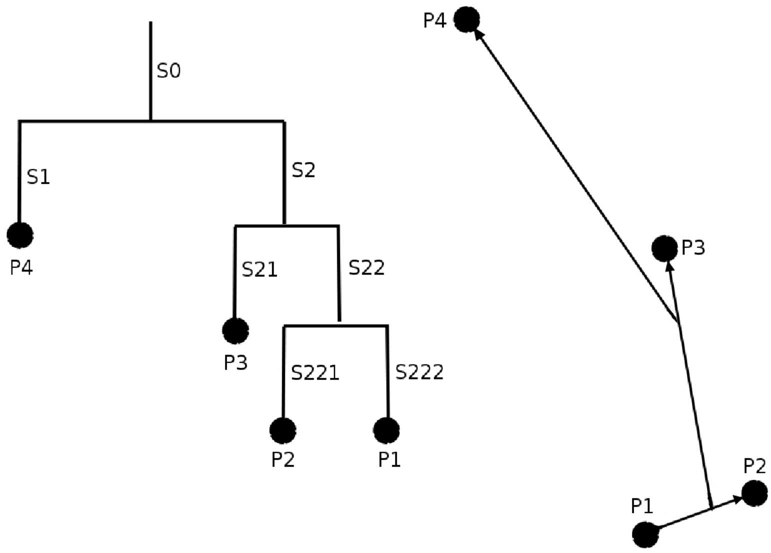

FIGURE I.13: Right hand image: hierarchical quadruple system with a double subsystem (P_1+P_2), and the third and fourth components at successively larger distances (P_3 and P_4). Left hand image: schematics of the hierarchy.

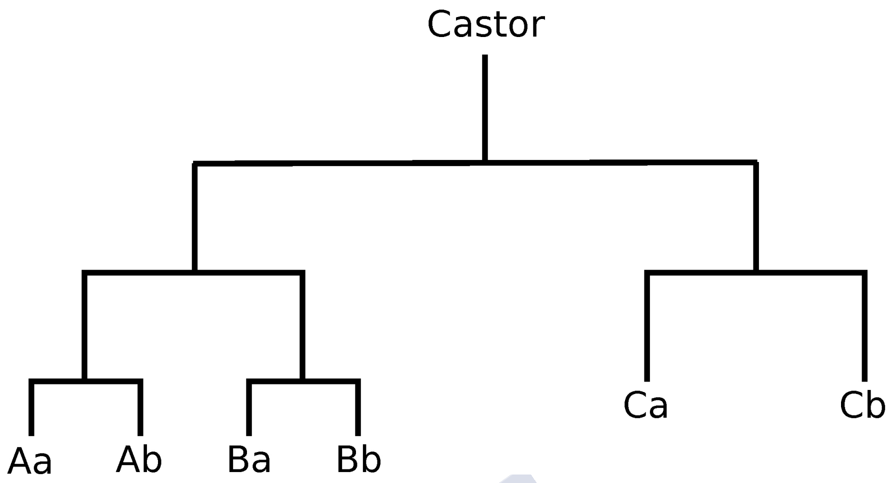

FIGURE I.14: Schematics of the hierarchy of the sextuple system Castor.

Throughout time, the dynamics of these systems have been studied in multiple ways focusing, for example, on the stability (Eggleton and Kiseleva, 1995; Georgakarakos, 2008, 2013; Grishin et al., 2017; Li, Fu, and Sun, 2010; Martynova, Orlov, and Rubinov, 2009; Milani and Nobili, 1983; Mylläri et al., 2018; Széll, Steves, and Érdi, 2004; Walker, 1983a,b; Walker and Roy, 1981, 1983a,b) or on their secular evolution under the effect of different conditions and perturbations such as the existence of coplanarity or not, and the influence of the inclination and the eccentricity, including the Lidov-Kozai effect (Georgakarakos, 2002, 2003, 2004, 2009; Grishin, Perets, and Fragione, 2018; Naoz et al., 2017), relativistic perturbations (Will, 2017), mass-loss and mass-transfer (Michaely and Perets, 2014), rotation and tidal effects (Borkovits, Forgács-Dajka, and Regály, 2004, 2007; Correia et al., 2011; Hamers, 2019), or the influence of the evolutionary state of the components (Toonen, Hamers, and Portegies Zwart, 2016). Many Hamiltonian formulations have developed different orders of perturbation, using different sets of coordinates (Breiter and Vokrouhlický, 2015; Ford, Kozinsky, and Rasio, 2000; Hamers and Portegies Zwart, 2016; Krymolowski and Mazeh, 1999; Lei, Circi, and Ortore, 2018; Naoz et al., 2013), and even specific numerical algorithms have been developed (Beust, 2003).

Sophisticated mathematical tools, both analytical and numerical, are required for the long-term study of hierarchical systems. However, in order to determine the masses of the stars for a short term study, the system can be decomposed into independent two-body problems because the mutual influence between distant subsystems is very small. If the difference between the scales of the distances of the different subsystems is large enough, we can calculate a Keplerian orbit for each subsystem without further correction as is the case of the star, Castor (Docobo et al., 2016), a hierarchical sextuple system with the schematics shown in Figure I.14. In this system, for example, the distance between the Aa and Ab subcomponents is 0.127 a.u.., between Ba and Bb, 0.0562 a.u., and between the subsystems A (Aa-Ab) and B (Ba-Bb), 105 a.u.

In other cases, corrections may be necessary such as the case of the Gliese 22 system (Docobo et al., 2008a), a hierarchical triple system for which some measurements did not resolve the inner pair and the position of the third component (B) was referred to the light center of the Aa-Ab subsystem, whereas the most modern observations did resolve it and the B component was measured with respect to Aa and it was necessary to homogenize them in order to calculate the relative orbit of B with respect to Aa-Ab.

There are studies of individual visual, spectroscopic, and eclipsing multiple systems (Borkovits et al., 2019; Catanzaro et al., 2019; Docobo and Andrade, 2006; Horch et al., 2019; Jha et al., 1997; O'Brien et al., 2011; Shultz et al., 2019; Tokovinin, 2016a,b) and A. Tokovinin (Tokovinin, 2018) has produced a catalog of known hierarchical systems.

Chapter II

Extrasolar planets

As we have seen in the Introduction, the study of extrasolar planets, or exoplanets, is a field that is growing rapidly within Astronomy. Besides the constant discoveries, many studies are being conducted regarding the dynamics of different scenarios involving exoplanets. In this Chapter, we will review this field of research beginning with their classification. We will then review the observation techniques used for the discovery and the study of the extrasolar planets. Next, we will inspect the research concerning the dynamics of planetary systems, focusing on binaries with exoplanets. Later, we will examine the search for exosatellites orbiting around these planets, reviewing the research, and we will present an original work in which we analyze the dynamics of planet-planet and planet-satellite systems in order to obtain an algorithm that permits the detection of exosatellites through the perturbations that they induce in the spectroscopic orbit of their host planet. Finally we will deal with the study of the habitability of exoplanets and exosatellites, meaning the possibility that liquid water exists on the surface of these bodies, with the goal of finding places with conditions similar to those on Earth.

II.1 Classification

As it happens in the Solar System, extrasolar planets with different masses and sizes are being discovered. Two types of planets were traditionally considered orbiting the Sun, the Earth-type (or rocky) planets, and the Jupiter-type (or gas giant) planets. Most of the extrasolar planets discovered could be assigned to one of those categories, however, it is not so clear in other cases. That is why several schemes of classification have been proposed over time, both formal and informal.

A first option is to use the planetary mass as a criterium for classification. Following this philosophy, S. A. Stern and H. F. Levison (Stern and Levison, 2002) proposed an expansive classification for planets, that in the case of the Solar System, would also include some minor bodies. According to their definition, the planets would be bodies with enough mass so that their shape was determined by gravity in a time lower than the Hubble time and their interior acquired hydrostatic equilibrium, but their mass would not be enough to start nuclear reaction in their nuclei which would exclude the brown dwarfs and the stars. In this way, there would be subdwarf planets with a mass less than 0.03 Earth masses ($\mathcal{M}_{\oplus}$); dwarf planets, up to $\mathcal{M}_{\oplus}$; subgigant planets, which would be limited by 100 $\mathcal{M}_{\oplus}$; the giant planets with masses less than 1000 $\mathcal{M}_{\oplus}$; and the supergiant planets with an upper limit set by the brown dwarfs. Another mass-based classification was proposed by the Planet Habitability Laboratory of the University of Puerto Rico at Arecibo (http://phl.upr.edu/library/notes/amassclassificationforbothsolarandextrasolarplanets). It considers 3 large groups defined by their mass. First, we have the group of the minor planets, asteroids, and comets in which they distinguish the asteroidans, with a mass less than 0.00001 $\mathcal{M}_{\oplus}$ and the mercurians, up to 0.1 $\mathcal{M}_{\oplus}$. The next group includes the terrestrial planets which are divided in subterran (0.5 $\mathcal{M}_{\oplus}$), terran (2 $\mathcal{M}_{\oplus}$) and superterran (10 $\mathcal{M}_{\oplus}$). Finally, there are the gas giants and,among them are the neptunian planets (50 $\mathcal{M}_{\oplus}$) and the jovian planets (5000 $\mathcal{M}_{\oplus}$).

Another possible criterium for the classification is the radius of the planet, as in the proposal by L. Zeng, S. B. Jacobsen, D. D. Sasselov,and A. Vanderburg (Zeng et al., 2018) that distinguishes among rocky planets with a radius less than 2 Earth radii ($R_{\oplus}$); aquatic planets, between 2 and 4 $R_{\oplus}$; transitional planets, with radii between 4 and 10 $R_{\oplus}$; and gas giants, with radii greaterer than 10 $R_{\oplus}$.

D. Sudarsky, A. Burrows, and P. Pinto (Sudarsky, Burrows, and Pinto, 2000) proposed a classification of gas giants according to their surface composition which is related to the surface temperature. First, we have Class I planets with a predominance of methane and ammonia clouds, and temperatures lower than 200 K. Class II planets are covered in water vapor and methane clouds and their temperature can reach 300 K. With temperatures between 350 K and 800 K, planets do not form clouds and they are included in Class III, methane being the main component of the atmosphere. Class IV planets register between 900 K and 1500 K and their outer atmosphere contains many alkaline metals. In Class V, we find planets with temperatures over 1500 K and clouds of silicates and alkaline metals.

The gas giants are divided according to the distance to the star because, although these planets form at moderate distances from their host star, some of them migrate, moving to a distance from the star less than that of Mercury from the Sun. They are called hot Jupiters or hot Neptunes. If the planet gets too close to the star, it can lose its atmosphere and the rocky core is exposed (Leitzinger et al., 2011). The name, cthonian has been proposed for these planets.

E. Plávalová (Plávalová, 2012) proposed a classification scheme based on the MK system for the stars. It includes four parameters: the mass given in Mercury (M), Earth (E), Neptune (N), or Jupiter (J) mass units; the decimal logarithm of their semimajor axis in astronomical units; the temperature associated with 4 classes: F (Freezing class), W (Water Class), G (Gaseous Class), and R (Roasters Class); and a Class P (Pulsar class) intended for planets

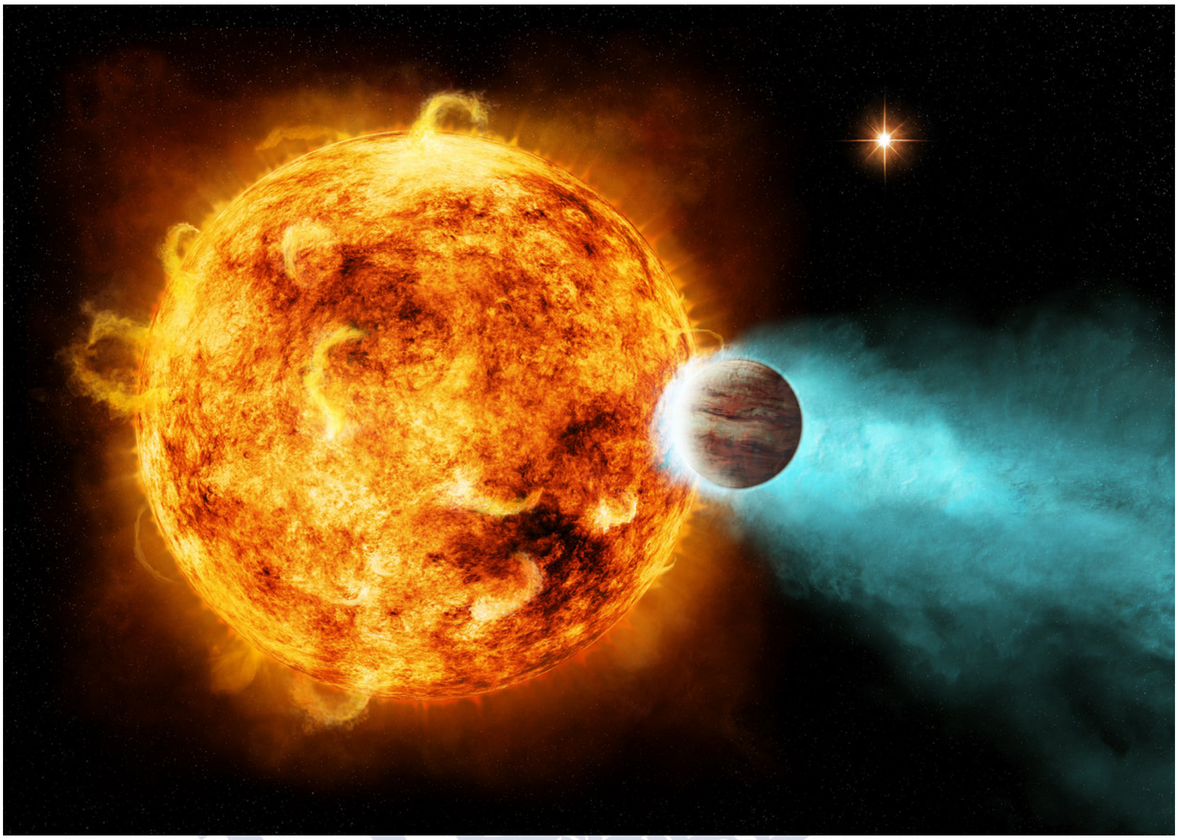

FIGURE II.1: Artistic representation of a hot Jupiter. Image: NASA/Ames/JPL-Caltech.

around pulsars; as well as the orbital eccentricity. If the surface conditions are known, a fifth parameter is included, with "t" representing a terrestrial planet, "g" representing a gas planet, and "I" representing an icy planet. Therefore, a 1E0W0t planet would be a rocky planet with 1 Earth mass at a distance of 1 a.u., with an orbital eccentricity of 0, and with a temperature compatible with liquid water.

II.2 Observation techniques

The biggest difficulty within the framework of extrasolar planet observation is that they are much smaller than the stars which they orbit and that they do not emit their own light. For example, even in the most favorable conditions, i.e., with its illuminated half oriented toward the observer, the luminosity that would reach us from a planet like Jupiter would be on the order of 10^{-9} times the Sun luminosity. That is why, until recently, their detection was limited by the sensitivity of the reception devices. In fact, many of the techniques that are used for their discovery and study come directly from double stars research, as we will see now.

II.2.1 Radial velocities

This is the same technique that is used for the study of spectroscopic binaries and it was used to detect the first planets orbiting a main-sequence star. The first difference is that, due to the magnitude difference between the star and the planet, we are not able to observe the spectrum of this last object. Whereas there are double-lined spectroscopic binaries, we can only observe the effect that the exoplanets cause in the radial velocity of the star.

Another important difference comes from the magnitudes of the radial velocities because in the case of spectroscopic binaries in general, they are measured in km s^{-1}, whereas for extrasolar planets, they are in the range of m s^{-1} or even less. For instance, the velocity of Jupiter would be around 12.5 m s^{-1} and, for the Earth, it would be 0.09 m s^{-1} (Perryman, 2018). This requires the use of spectrographs with ultra high resolution in order to be able to detect exoplanets such as HARPS (Mayor et al., 2003) which works in the visual band, and NIRPS in infrared, that are installed in the 3.6 m. telescope of La Silla Observatory (ESO, Chile), and their couterparts HARPS-N and GIANO-B, which operate from the 3.58 m. Telescopio Nazionale Galileo, in Roque de los Muchachos (Canarias, Spain). HIRES (Vogt et al., 1994) is an optical spectrograph and one of the instruments of the Keck I telescope with an aperture of 10 m. which is located in Mauna Kea (Hawaii, EEUU). ESPRESSO (Pepe et al., 2010) is considered the succesor of HARPS and it is installed in the VLTI in Cerro Paranal (ESO, Chile). CARMENES (Quirrenbach et al., 2016) works both in optic and infrared regimes and operates from the 3.6 m. telescope of CAHA (AlmerÃa, EspaÃ±a). All of these spectrographs are of the "echelle" type, and their gratings (echelle gratings, see figure II.2) have a high difraction order. They separate further the spectral features and therefore yield larger resolutions (Michelson, 1898) with values of R in the range of 80000 to 115000, where $R = \frac{\lambda}{\Delta\lambda}$, with λ being the wavelength and $\Delta\lambda$ being the smallest detail that can be distinguished at that wavelength. These resolutions permit the measurement of radial velocities on the order of 1 m s^{-1} and it is expected that ESPRESSO, which has started to work, reaches velocities of cm s^{-1} when it operates from the interferometer of the VLT.

When we work with such small radial velocities, we cannot neglect other influences like the ones that we commented on the case of the spectroscopic binaries, and they must be incorporated into the model. Thus, apart from the variations caused by the movements of the Earth and secular phenomena, the rotation of the star or the presence of star spots may affect the measurement.

In many cases, there may be more than one planet in the system and the signal of several of them may be above the detection threshold. In that case, several movements would appear superimposed in the radial velocity curve of the star. Except for very rare occasions, the

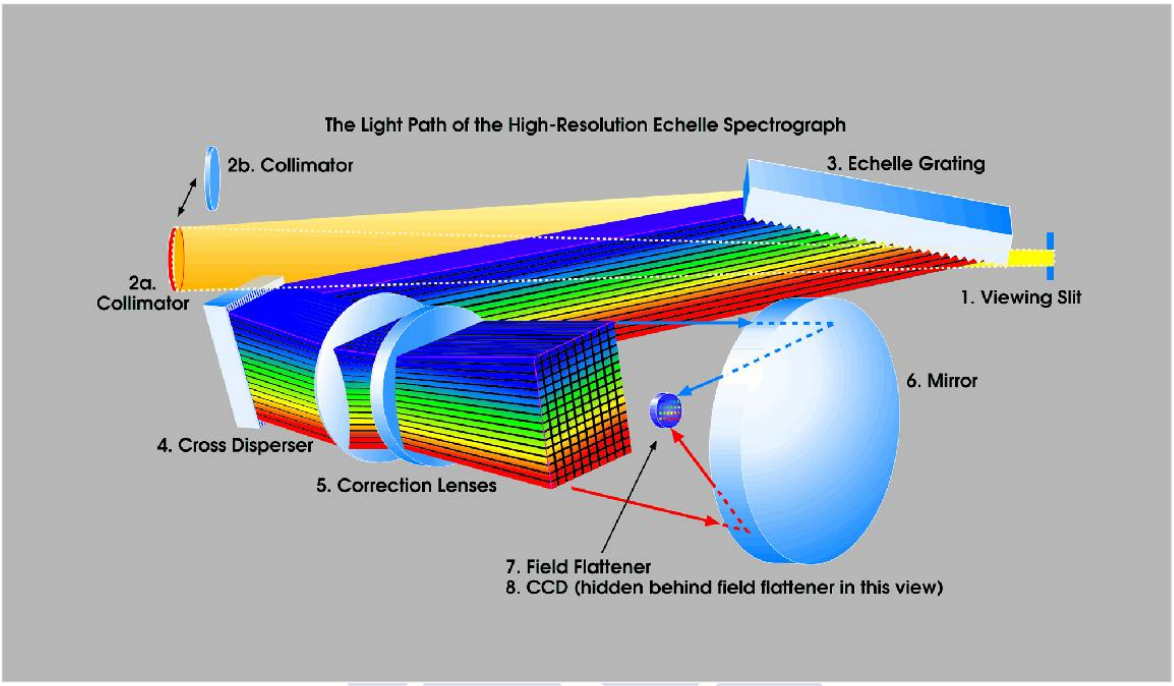

FIGURE II.2: Diagram of an echelle spectrograph. Image: HIRES/Keck.

amplitudes of the signal of each planet will have different magnitude and, unless there are significative interactions among them, we can determine them as Keplerian movements and eliminate them succesively from the radial velocity curve. If the Keplerian fit is not good, for the long term dynamical study of the system it is necessary to solve a n-body problem analytically, if it is possible, or numerically, in other cases (Perryman, 2018).

II.2.2 Transits

The transits technique comes directly from the study of eclipsing binaries. It consists in measuring the decrease in the light received from the star when the planet passes in front of it (transit). The difference with the eclipsing binaries is that the planet is much smaller than the star, therefore, it will never cover the star completely nor even a significant part of it. In addition to this, when the planet passes behind the star, the decrease in luminostity is even smaller. That is why, contrary to binaries, we will usually not have two different periodical minima but a single minimum of very small magnitude. Only by using the most sensitive photometers we are able to detect the eclipse of the planet as well as the increase in the brightness caused by the phase of the planet because, when it is farther from us than the star, a larger portion of the illuminated hemisphere is oriented toward the Earth and viceversa (see image II.3). The models for this case are a direct adaptation of the ones used for eclipsing binaries, except for the contact cases that do not occur in this context.

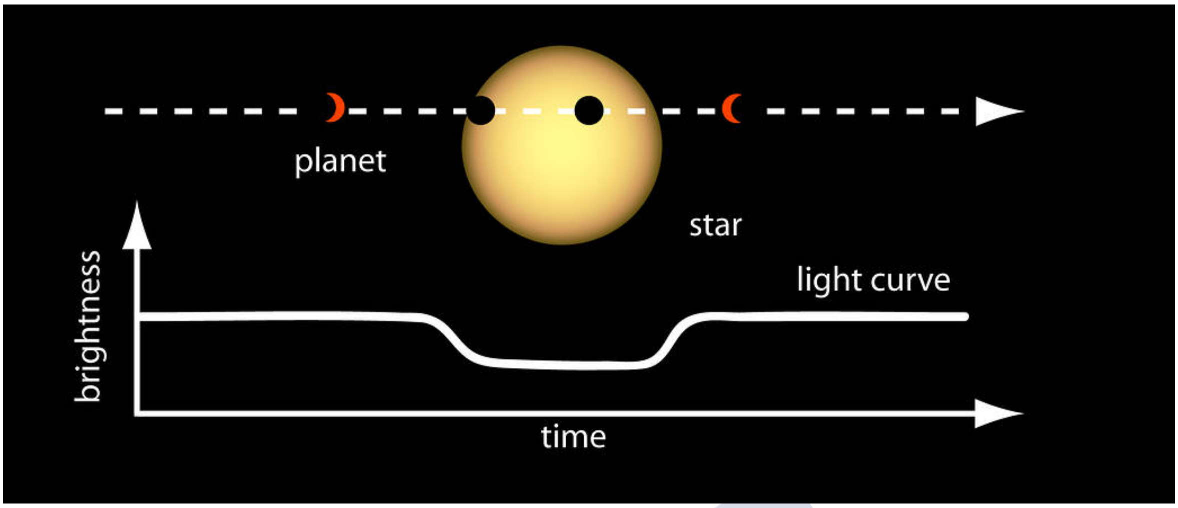

FIGURE II.3: Transit schematic. Image: NASA

In the beginning, this technique was less effective than that of the radial velocities in the number of detections, but gradually and thanks to dedicated space telescopes like the CoRoT or Kepler, it has established itself as the most successful with thousands of discoveries. The transits have several advantages over radial velocities. They are less limited by the size of the planet and, even in some cases, signals of the atmosphere of the planet could appear in the spectrum of the star if it was measured during a transit. Besides, it is desirable to detect the exoplanets with several techniques when possible, and many planets that are discovered by means of radial velocities are confirmed by transits and viceversa. The greatest difficulty is that the geometry of the system must have a certain geometry for the transit to occur. The probability that a planet with an arbitrary orientation transits in front of its star is given by the following expression (Borucki and Summers, 1984):

$$p = \frac{R_*}{a} \tag{II.1}$$

where R_* represents the radius of the star and a is the semimajor axis of the planetary orbit. The probability is very low but, due to the huge number of stars in our Galaxy, there are enough planets that transit in front of their stars to achieve numerous discoveries.

Due to the successes obtained by this technique, there are multiple observation programs in operation, both from space and from the Earth, and many others are in preparation. Regarding the space telescopes, since the end of the Kepler mission in November 2018, there is a new space telescope devoted to this technique, the Transiting Exoplanet Survey Satellite (TESS) of NASA which was launched in April 2018 and saw its first light in August of the same year (Ricker et al., 2015). Contary to the Kepler, the TESS is not limited to a small portion of the sky. In October-November 2019, another space telescope will be launched, the CHaracterising

ExOPlanets Satellite (CHEOPS) of ESA with the main mission of measuring the radii of the planets with high precision (Broeg et al., 2013). In 2021 the launch of the James Webb Space Telescope (JWST) is scheduled which will be the largest telescope in orbit with an aperture of 6.5 m. of diameter. Among its functions will be the detailed study of extrasolar planets and the characterization of their atmospheres (Gardner et al., 2006). In 2026, ESA plans to put into orbit another space observatory, the PLAnetary Transits and Oscillations of stars (PLATO) dedicated to exoplanets, with the objective of observing up to a million stars searching for Earth-type and SuperEarth-type planets in habitable zones (Rauer et al., 2016). Among the land programs, the Wide Angle Search for Planets (SuperWASP) operates at the Roque de los Muchachos Observatory on La Palma island, and from Southafrica (Butters et al., 2010). There is also the XO Project (McCullough et al., 2005) that today has three cameras installed at the Vermillion Cliffs Observatory (Utah, USA), at the Teide Observatory (Tenerife, Spain), and at the Montsec Astronomical Observatory (Lleida, Spain). At this last Observatory, we can also find the Fabra-ROA telescope that is able to detect transiting exoplanets (Fors et al., 2010). The Hungarian Automated Telescope Network (HATNet) operates from Kitt Peak (Arizona, USA) in the Northern Hemisphere, and from Australia, Namibia, and Chile in the Southern Hemisphere (Bakos et al., 2002). The Kilodegree Extremely Little Telescope (KELT) has two observing stations, one in Arizona (USA) for the Northern Hemisphere and another in Sutherland (Southafrica).

Just like the case of eclipsing binaries, the study of exoplanets by means of transits permits us to obtain some of the orbital elements such as the period, the inclination, and the eccentricity, as well as the ratio between the raddi of the planet and the star or the ratio between the semimajor axis and the radius of the star.

II.2.3 Imaging

It has already been established that the brightness of a planet is much less than that of the star around which it orbits. Numerically, the ratio between the flux of the planet, f_p, and the flux of the star, f_*, for a certain wavelength, λ, is given by the expression (Perryman, 2018):

$$\frac{f_p(\alpha, \lambda)}{f_*(\lambda)} = p(\lambda)g(\alpha)\left(\frac{R_p}{a}\right)^2 \tag{II.2}$$

Here $p(\lambda)$ represents the geometric albedo of the planet in that wavelength, R_p stands for the planet radius, and a indicates the semimajor axis of the orbit. It must be noted that the flux of the star only depends on the wavelength, whereas for the planet, we must take into account the phase included within the parameter α as follows:

$$cos\ \alpha = -sin\ I\ sin(2\pi\phi) \tag{II.3}$$

where I represents the inclination of the orbit and $\phi \in [0, 1]$ is the phase of the planet, with a value of 0 when it reflects all of the light in the opposite direction from the Earth and 1 when the hemisphere of the planet that we see is completely illuminated. The function $g(\alpha)$ is defined as follows:

$$g(\alpha) = \frac{sin\ \alpha + (\pi - \alpha) cos\ \alpha}{\pi} \tag{II.4}$$

In addition to the difference in magnitude, another problem arises which is that, on many occasions, the planet as it is observed from the Earth is located inside the seeing disc of the star, obfuscating the signal of the planet. Even near the theoretical difraction limit of the telescope, the presence of the concentric rings of the Airy function may render the observation impossible. High resolution techniques like the ones commented in the section about visual binaries can be helpful, although they do not currently have the detction power of the transit method or the radial velocities. The development of larger telescopes, the European Extremely Large Telescope (E-ELT), for example, will increase the number of discoveries, as will the use of space telescopes like the JWST.

However, there are techniques that permit the improvement of detection with the current telescopes. First, we have the use of coronographs that were originally developed for the observation of the solar corona (Lyot, 1939), as their name indicates. The classic coronography (also known as Lyot coronography) consists in incorporating a mask on the telescope axis in the way that it blocks the light from the central object, improving the SNR of the bodies located off-axis. In the case of exoplanets, the coronography alone is not enough to achieve their detection but when it is combined with adaptive optics (Malbet, 1996; Sivaramakrishnan et al., 2001), it has yielded positive results. There are many other designs of coronographs that are based not only on physical masks but also on the use of destructive self-interference by means of phase shifts (Guyon et al., 2006).

There are also methods to reduce the noise of the image that can be applied along with the coronography. We have, for example, Angular Differential Imaging (ADI) which is based on taking a large quantity of short-exposure images (Marois et al., 2006). The device to correct for the paralactic angle is turned off in the telescopes that have it in order to improve the quality of the image. For each image, a synthetic PSF of the star is built from the other observations taking into account that they must be aligned to correct the rotation of the field of view. The synthetic PSF is substracted from the image and that permits the elimination of the quasi-static speckles, reducing the noise level in ~5. Other technique is Simultaneous Spectral Differential Imaging (SSDI) that consists of taking two simultaneous images with two different narrow-band filters centered on close wavelengths (Racine et al., 1999). ADI and SSDI can be combined with other techniques to further improve the SNR, such as LOCI (Lafrenière et al., 2007), TLOCI (Marois et al., 2014), or SOSIE (Marois, Macintosh, and Véran, 2010).

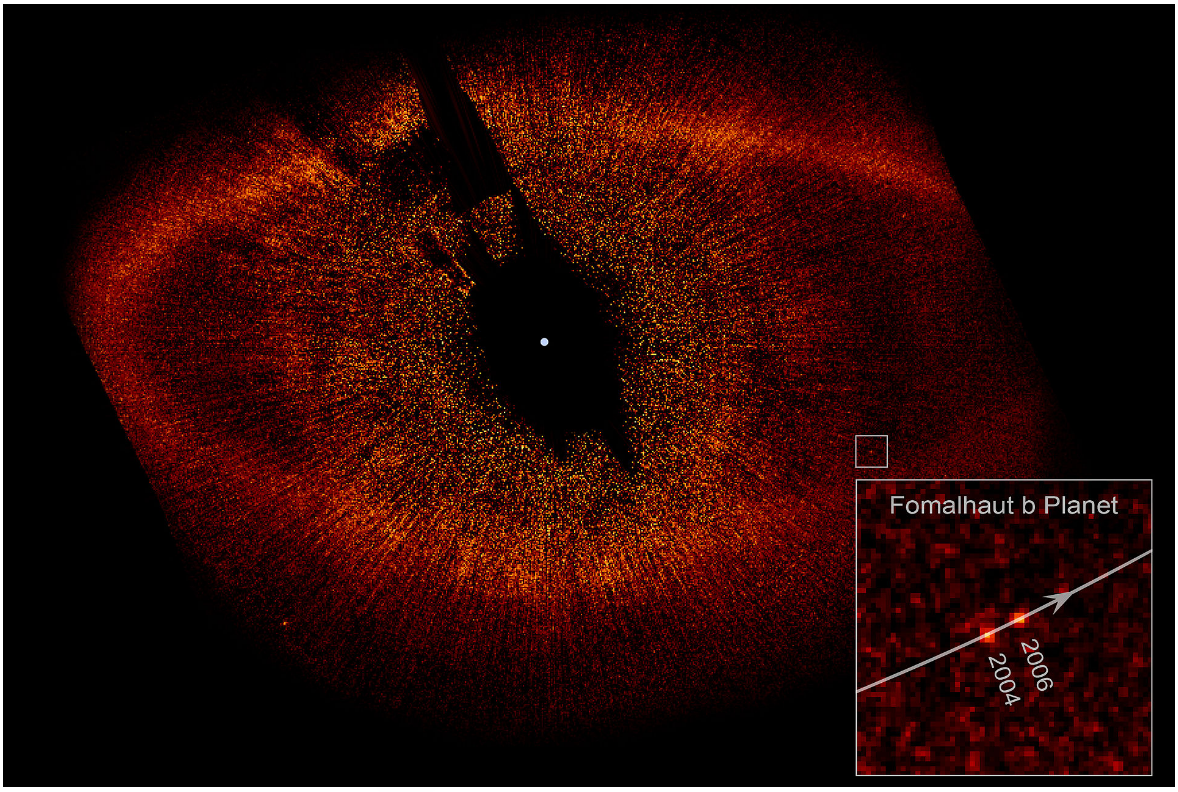

FIGURE II.4: Planet orbiting the star, Fomalhaut (α PsA). Image: NASA, ESA, P. Kalas, J. Graham, E. Chiang, E. Kite (University of California, Berkeley), M. Clampin (NASA Goddard Space Flight Center), M. Fitzgerald (Lawrence Livermore National Laboratory), K. Stapelfeldt and J. Krist (NASA Jet Propulsion Laboratory)

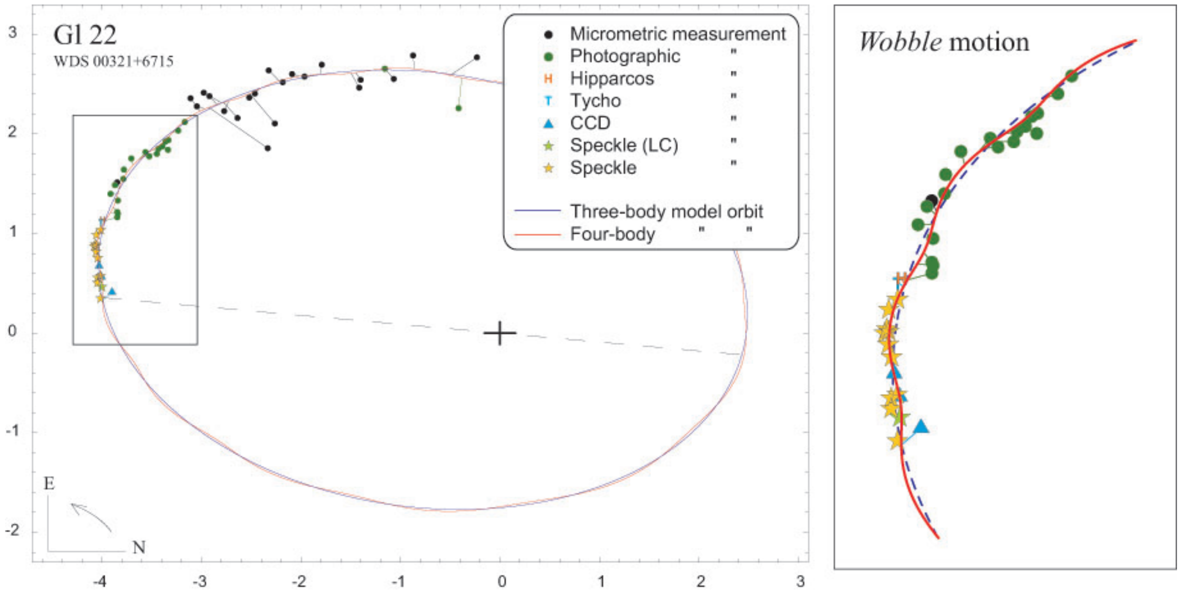

FIGURE II.5: Outer orbit of the system Gl22.

II.2.4 Astrometry

We have seen that a subset of the visual binaries is the astrometric pairs. In that type of system, due to the difference in magnitude, we could only observe the light of the primary which underwent variations of its observed position because of its movement around the center of mass. This is the same case of a star with a planet around it but at a much smaller scale. If a represents the semimajor axis of the relative orbit of the planet around the star, the semimajor axis of the orbit of the star with respect to the baricenter can be calculated from:

$$\alpha = \frac{M_p}{M_p + M_*} a \tag{II.5}$$

For example, if we consider a planet similar to Jupiter that is at a distance of 5 a.u. from a star with the mass of the Sun and at 10 pc. from the observer, the calculated value is $\alpha \simeq 5 \cdot 10^{-4}$ arcseconds. Obviously, this technique is more suitable for massive planets around low mass stars as is the case of the Gliese 22 system (Docobo et al., 2008a), a hierarchical triple system of red dwarfs. This system showed, in the outermost orbit, an astrometric oscillation caused by the possible presence of an object of a mass around 16 $\mathcal{M}_{Jup}$. Therefore, it would be in the limit between giant planets and brown dwarfs (see image II.5).

For less massive planets, or planets closer to the star, we would have to achieve a precision of micro-arcseconds (μas) which is even lower than the limits of Gaia. At this scale, we need to take into account relativistic corrections of the light path caused by the mass of the Sun.

Even phenomena on the surface of the star such as star spots may produce variations in the position of the photocenter of that magnitude.

In order to obtain an astrometric measurement, we need to know other data that affect the position of the photocenter at a larger scale, specifically, the proper motion and the parallax of the system. If those are well determined with a large number of observations, the Campbell elements, P, T, e, a, I, Ω, and ω can be calculated by observing the variation of the equatorial coordinates of the photocenter. Concretely, we can obtain $\Delta\alpha \; cos \; \delta$ and $\Delta\delta$, with α and δ being the equatorial coordinates of the star. For a given instant, t, it verifies that (Wright and Howard, 2009):

$$\Delta\alpha(t) \; cos \; \delta = [BX + GY] + \Delta\alpha_0 \; cos \, \delta + \pi \; \Pi_{\alpha,t} + \mu_\alpha \; (t - t_0) \tag{II.6}$$

$$\Delta\delta(t) = [AX + FY] + \Delta\delta_0 + \pi \; \Pi_{\delta,t} + \mu_\delta \; (t - t_0) \tag{II.7}$$

Here, the Thiele-Innes constants A, B, F, and G are:

$$A = a \; (cos \, \omega \; cos \, \Omega - sin \, \omega \; sin \, \Omega \; cos \, i) \tag{II.8}$$

$$B = a \; (cos \, \omega \; sin \, \Omega + sin \, \omega \; cos \, \Omega \; cos \, i) \tag{II.9}$$

$$F = a \; (-sin \, \omega \; cos \, \Omega - cos \, \omega \; sin \, \Omega \; cos \, i) \tag{II.10}$$

$$G = a \; (-sin \, \omega \; sin \, \Omega + cos \, \omega \; cos \, \Omega \; cos \, i), \tag{II.11}$$

whereas X and Y are calculated from:

$$X = cos \, E(t) - e \tag{II.12}$$

$$Y = \sqrt{1 - e^2} \; sin \, E(t) \tag{II.13}$$

where $E(t)$ is the eccentric anomaly in the instant t. π in the equations II.6 and II.7 represents the parallax, whereas $\Pi_{\alpha,t}$ and $\Pi_{\delta,t}$ stand for the orthogonal displacements in α and δ due to the parallax, respectively. $\mu_\alpha(t - t_0)$ and $\mu_\delta(t - t_0)$ account for the displacement caused by the proper motion and $\Delta\alpha_0 cos\delta$ and $\Delta\delta_0$ indicate the difference in the coordinates of the star in t_0 with respect to the nominal coordinates. It has to be noted that the equations, II.6 and II.7, permit the modelling of multiple planet systems by including the terms $[BX + GY]$ and $[AX + FY]$, for each planet.

II.2.5 Timing

We call "timing" the discovery of an exoplanet due to the variation that it produces in other periodical phenomenon such as another exoplanet previously detected by means of transits, an eclipsing binary, or periodical pulsations of the star. Due to the orbit that the star describes with respect to the center of mass of the star-planet system, the path that the light traverses is longer when it is in the farthest part of the orbit. This causes a delay in the observed moment of the periodical phenomenon or an advance when the star is at the nearest point. Numerically, the amplitude of this variation is given by the following expression:

$$\tau = \frac{1}{c} \mathcal{M}_p \ a \ sin I \mathcal{M}_* \tag{II.14}$$

where c represents the speed of light in the vacuum, $\mathcal{M}_p$ and $\mathcal{M}_*$ indicate the masses of the planet and the star, respectively, and a and I stand for the semimajor axis and the inclination of the orbit of the planet relative to the star. Obviously, this time difference will be very small, therefore we need highly accurate values of the perturbed phenomenon in order to discover exoplanets with this technique.

II.2.6 Pulsar Timing

One type of periodical phenomena in which we can observe the variations due to the presence of exoplanets is the pulsars. These objects are neutron stars, i. e., the remains of the nucleus of a massive star (> 9 $\mathcal{M}_\odot$) that exploded as a supernova. Neutron stars have a typical mass of approximately 1.4 $\mathcal{M}_\odot$ and the gravitational attraction forces the matter to condense to a few km. of diameter. Due to the gravitational pressure, the matter inside these objects degenerates and the protons and the electrons combine, forming a superfluid of neutrons. The conservation of the angular moment causes the rotation of these objects to speed up when the radius diminishes, reaching periods of miliseconds. Neutron stars develop strong magnetic fields that produce the emission of radiation (γ rays, X rays, visible, and radio) from the magnetic poles. Usually, the magnetic and the rotation axes don't match and consequently, when the neutron star rotates, the beams of radiation sweep an area of the sky. If that area contains the Earth, periodic pulses of radiation will reach our planet and the neutron star is known as a pulsar.

These objects were predicted by Baade and Zwicky in 1933 (Baade and Zwicky, 1934) although their prediction did not receive much attention in that time. It was in 1968 when the first radio pulses from one of these objects were detected in the Mullard Radio Astronomy Observatory by Jocelyn Bell and Antony Hewish (Hewish et al., 1968). The rotational period of the pulsars is highly stable although it decays very slowly due to the radiation and the

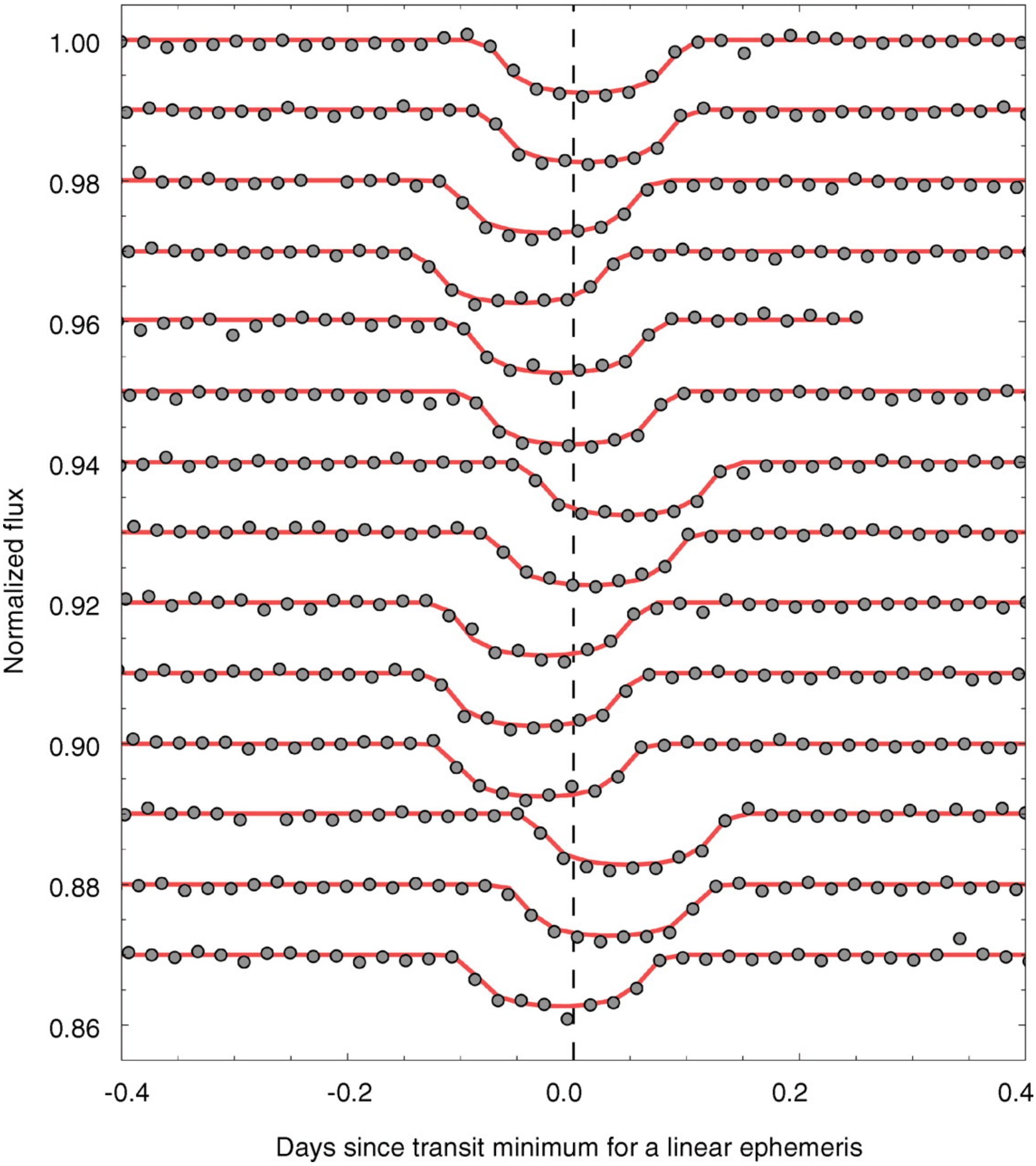

FIGURE II.6: Variations in the transits of KOI-872b. Image: included in Nesvorný et al. (2012)

magnetically accelerated particles that it emits. This decay follows the formula (Burke and Graham-Smith, 2009):

$$\frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = \frac{2}{3}M_\perp^2\omega^4c^{-3} \tag{II.15}$$

Here ω represents the angular velocity, I indicates themoment of inertia, and $M_\perp$ stands for the component of the dipolar magnetic moment that is orthogonal to the rotation axis. The diminution of the rotational velocity is very small with tiny occasional increments that are called "glitches". After a long time, when the period of rotation reaches several seconds, the spin of the magnetic field is no longer able to feed the radio emission and the pulsar is not observable anymore.

If the period of ration of the pulsar shows faster periodical changes, these may indicate the presence of other bodies in the system, for example, planets (see image II.7). The regularity of this movement and its short period make these exoplanets easy to detect even if their mass is as low as 0.020 $\mathcal{M}_\oplus$. This is the case of Draugr (PSR B1257+12 b) which belongs to the system in which the first confirmed exoplanets were discovered (Wolszczan, 1994). The difficulties arise from the relatively small number of detected pulsars as well as the fact that the planetary systems in the original star are likely destroyed during the supernova phase that produces the pulsar, therefore the exoplanets must form after the explosion with the remaining material that surrounds the nucleus.

II.2.7 Microlensing

The Theory of General Relativity predicts that a massive object curves the space-time around it and, consequently, if there is another luminous object behind it, the path of the light rays emitted by the most distant object will be distorted thereby yielding the phenomenon of the "gravitational lens". It is called "macrolensing" when we can resolve the distorted image of the farthest object and deformed ghost images or arcs appear around the closest object. In the case of "microlensing", the image cannot be resolved, and it is detected as an increase in the brightness of the distant object due to the light rays being focused toward the observer. If the closest object is a star, the presence of an exoplanet around it will cause a secondary peak of smaller magnitude in the light curve observed during the microlensing, thus allowing its detection (see image II.8).

This technique has two disadvantages. First, the planet can only be detected during the microlensing which is a one-time event and other methods are needed for a follow-up study. Moreover, the probability of a microlensing event is low, therefore the microlensing observation programs are focused on the observation of the regions of the Galaxy toward the core where the concentration of stars is higher. On the other hand, the observation of microlensing

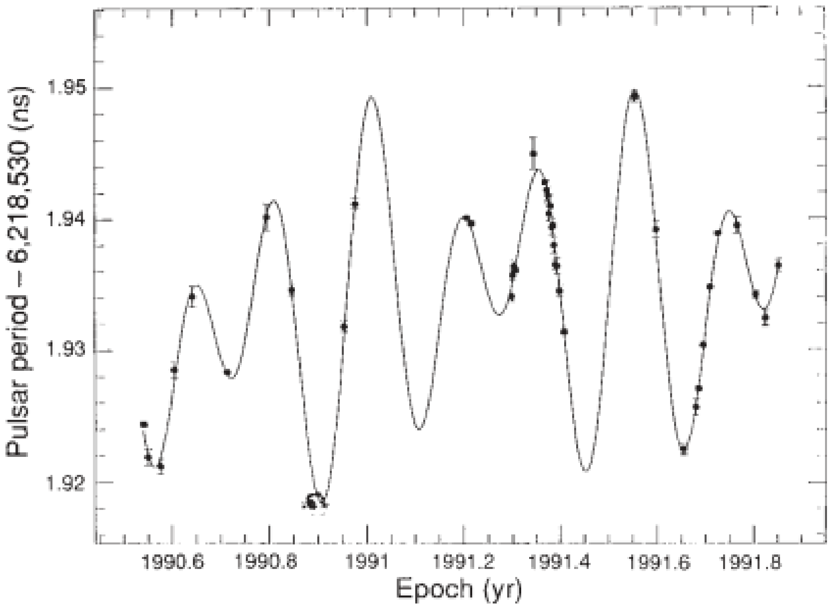

FIGURE II.7: Variations in the period of rotation of the pulsar PSR B1257+12. Image: A. Wolszczan, D. Frail

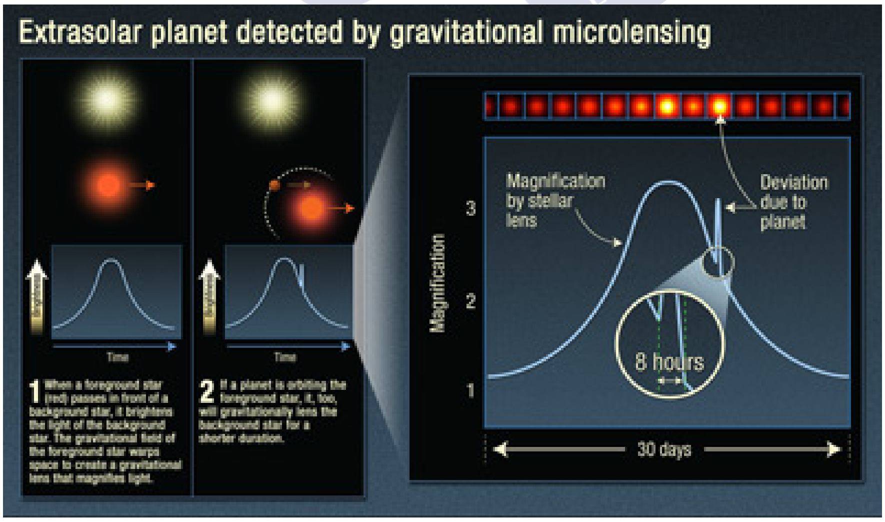

FIGURE II.8: Schematic of the detection of an exoplanet by means of microlensing. Image: NASA, ESA, and A. Feild (STScI) is less limited by the distance than other techniques and it has been able to detect planets at distances up to 6500 pc.

II.3 Dynamics of extrasolar planets

From a mathematical point of view, there are also many problems to study. The dynamics of exoplanets, both in single stars and in double and multiple systems, has opened an extraordinary field of research in Astrodynamics. Among others, there are questions about the study of co-orbital motions, resonances and their influence in the variation of the eccentricity, the stability of orbits, and orbital and rotational perturbations. They have attracted the attention of renowned specialists in Celestial Mechanics, some of them before the discovery of the first exoplanet. Examples of these works are: Beaugé, Ferraz-Mello, and Michtchenko (2003), Beaugé, Michtchenko, and Ferraz-Mello (2006), Callegari, Michtchenko, and Ferraz-Mello (2004), Callegari, Ferraz-Mello, and Michtchenko (2006), Dvorak (1980, 1984, 2006), Dvorak, Froeschle, and Froeschle (1989), Dvorak and Henrard (1988), Dvorak and Süli (2002), Dvorak et al. (2010), Ferraz-Mello, Beaugé, and Michtchenko (2003), Ferraz-Mello and Michtchenko (2002), Ferraz-Mello (2014, 2015), Funk et al. (2004, 2011), Funk, Dvorak, and Schwarz (2013), Funk et al. (2009), Giuppone et al. (2010), Hagel and Dvorak (1987), Kubala, Black, and Szebehely (1993), Michtchenko, Beaugé, and Ferraz-Mello (2006, 2008a,b), Michtchenko, Ferraz-Mello, and Beaugé (2006), Nesvorný et al. (2002), Pilat-Lohinger and Dvorak (2002), Pilat-Lohinger, Funk, and Dvorak (2003), Rabl and Dvorak (1988), Rodríguez et al. (2011), Szebehely (1979, 1980), Szebehely, Black, and Kubala (1995), Szebehely and McKenzie (1977, 1981), Szebehely (1967), and Tadeu dos Santos et al. (2015), among others.

As a first approximation for the study of the dynamics of planetary systems in single or multiple stars, we can suppose that the movement of the star or stars of the system is not affected by the gravitational force of the surrounding planets. In 1984, A. L. Whipple and V. Szebehely (Whipple and Szebehely, 1984) published an article in which they described the restricted $n + \nu$ body problem (n bodies, not necessarily point-like, each one of them with a considerable mass, that are called primaries and ν infinitesimal masses which do not affect the primaries) interacting among them by means of arbitrary forces (not only gravity). For $n = \nu = 1$ and point masses, we have the two-body problem whereas the case with $n = 2, \nu = 1$ is the classical restricted three-body problem which can occur, for instance, in a binary star with a planet. The values, $n = 1$ and $\nu = 2$, would yield a star with two planets, and $n = 2$, $\nu = 2$ create the so-called restricted 2+2-body problem, for which numerous equilibrium solutions have been obtained (Whipple, 1984), both around the colineal Lagrangian point and the triangular ones. In the cases of $n = 2, \nu = 1$ and $n = 2, \nu = 2$ the motion of the primaries is usually considered to be circular.

II.3.1 Dynamics of exoplanets in binary sistems

In a planetary system around a single star, there is ample tolerance for the existence of stable orbits. It suffices that the planets are sufficiently separated in order to avoid that their mutual gravitational attractions cause large perturbations in their quasi-Keplerian trajectories. Of course, they also have to verify Kepler's Third Law:

$$\frac{\mathcal{M}_* + \mathcal{M}_{P1}}{\mathcal{M}_* + \mathcal{M}_{P2}} = \frac{a_1^3/P_1^2}{a_2^3/P_2^2} \tag{II.16}$$

between each pair of planetary orbits. We use the following formalism:

- $\mathcal{M}_*$, the mass of the star.
- $\mathcal{M}_{Pi}$, the mass of each planet (i=1,2).
- a_i, the semimajor axis of the planet with mass $\mathcal{M}_i$ (i=1,2).
- P_i the corresponding orbital period. (i=1,2).

When one planet or more coexist in a double star, things are different. We know from the study of the restricted three-body problem (Battin, 1987; Danby, 1988; Szebehely, 1967) that stable orbits can only exist when the infinitesimal body either moves far away from the primaries or close to one of them or it is located in the Lagrangian points, L4 and L5. The extension of the areas of permitted stable motion depend on the masses of the primaries.

According to the criterium of R. Dvorak (Dvorak, 1984), The following types of stable orbits arise:

- planet-type orbits (P-type): when the exoplanet moves around both components of the binary system.
- satellite-type orbits (S-type): this is the case when the planet orbits one of the stars.
- libation-type orbits (L-type): if the planet is located in one of the lagrangian points.

The first case would correspond to a close binary, for example a spectroscopic binary with a planet moving around the center of mass of the stars (circumbinary planet). The second situation happens when the planet moves close to one of the components of a double star, usually a wide system such as a visual binary. At the moment, the planets detected in binaries are found in P-type orbits in 23 occasions, and in S-type orbits in 715 cases (Wright et al., 2011). The low number of P-type orbits is due mainly to the technical difficulties for their detection by means of both transits and radial velocities.

L-type orbits are expected to be very rare because, in order to be stable, the mass ratio between the components of the binary must be:

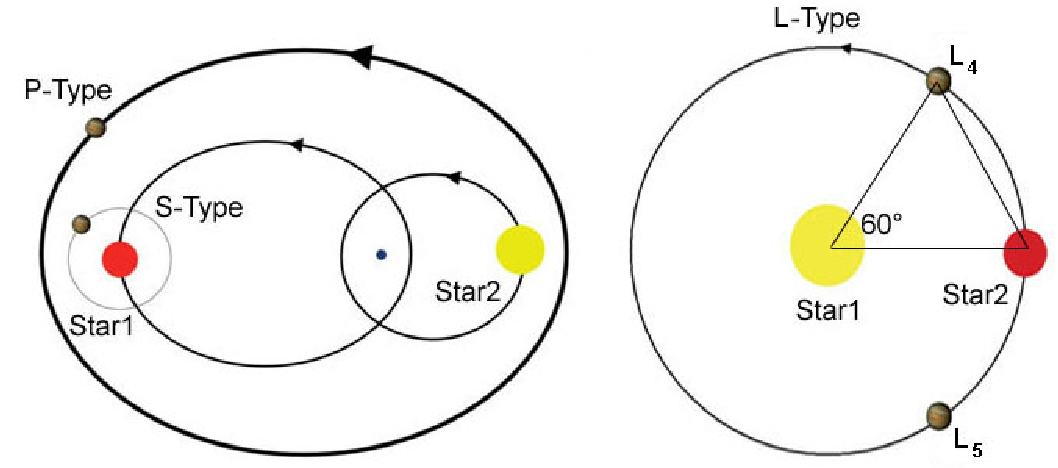

FIGURE II.9: Diagram of S, P, and L type orbits in binaries. Image: R. Schwarz.

$$\frac{M_1}{M_2} \geq \frac{25 + 3\sqrt{69}}{2} \approx 24.96 \tag{II.17}$$

which is extremely unusual for the range of stellar masses, although there is also the possibility of an Earth-type planet in this kind of configuration with a star and a giant planet (Dvorak, Schwarz, and Lhotka, 2008).

The stability of S-type and P-type orbits depends on the ratio between the semimajor axes of the orbit of the planet and the orbit of the binary. Limits have been established for the semimajor axis of the planetary orbit, an upper value in the case of S-type orbits (a_s), and a lower value for P-type orbits P (a_i). They are a function of the eccentricity and the semimajor axis of the binary star orbit (e and a, respectively) and of the ratio $\mu = \frac{\mathcal{M}_2}{\mathcal{M}_1+\mathcal{M}_2}$ ($\mathcal{M}_1 \geq \mathcal{M}_2$ are the masses of the components of the binary). In the case of planets with initially circular orbits, the values of a_s and a_i can be calculated by using the following expressions (Holman and Wiegert, 1999):

$$\begin{aligned}\frac{a_s}{a} = &(0.464 \pm 0.006) + (-0.380 \pm 0.010)\mu + (-0.631 \pm 0.034)e \\ &+ (0.586 \pm 0.061)\mu e + (0.150 \pm 0.041)e^2 + (-0.198 \pm 0.047)\mu e^2\end{aligned} \tag{II.18}$$

$$\frac{a_i}{a} = (1.60 \pm 0.04) + (4.12 \pm 0.09)\mu + (5.10 \pm 0.05)e + (-4.27 \pm 0.17)\mu e \\ + (-2.22 \pm 0.11)e^2 + (-5.09 \pm 0.11)\mu^2 + (4.61 \pm 0.36)\mu^2 e^2 \tag{II.19}$$

It is important to point out that in P-type orbits, there is also an upper limit of the semimajor axis of the planetary orbit that must be evaluated in each case taking into account the perturbations produced by nearby stars that may destabilize the orbit of the planet.

II.4 Exosatellites

As happens in our Solar System in which there are satellites of considerable size and some with an atmosphere, it is obviously fair to consider the existence of exosatellites or exomoons orbiting giant planets which may have the necessary conditions to harbor life and be similar to our Earth. Heller and Pudritz (2015a,b) proved that water ice lines could appear in the accretion disk of super-Jovian planets at distances of about 5 astronomical units from the star and that they could act as migration traps for forming moons and allow the existence of Mars-mass satellites around these planets.

There is considerable interest in the dynamical, physical, and astrobiological study of exomoons. Techniques such as the Transit Timing Variation (TTV) and Transit Duration Variation (TDV, see Heller, 2014; Kipping, 2009a,b; Kipping et al., 2012, 2013a,b, 2014, 2015) are considered ideal for detection. A few exomoon candidates have been postulated (Bennett et al., 2014; Kenworthy and Mamajek, 2015; Kislyakova et al., 2016) but the most promising is the recent Neptune-size companion to the super-Jovian Kepler-1625b as reported by Teachey, Kipping, and Schmitt (2018). One of the articles included in this Memory studies the dynamics of a binary star system with a planet and a satellite on a S-type orbit around one of the stars.

In this section we perform a comparative dynamical study of two different situations. In one of them, two planets orbit the host star whereas, in the other case, a planet with a satellite is moving around the star. In these scenarios we are interested in the short-term evolution of the system, that is why a long term integration is not necessary. The core idea is to study perturbations of the orbital elements of the main planet that are caused either by another planet or by a satellite. We will then see how these differences translate into the radial velocity signal with the aim of providing a methodology for the discovery of exomoons. A previous work of this kind was presented in Andrade and Docobo (2006b).

II.4.1 Equations of motion

In both cases, we have a central star with 1 $\mathcal{M}_\odot$ and a main planet whcihc will be a gas giant of 1 $\mathcal{M}_{Jupiter}$. Then we formulate them as three-body problems in order to compare and evaluate each situation. In case 1, we add a Neptune-size planet. In case 2, we add a satellite with 1 $\mathcal{M}_\oplus$.

Finally, simultaneous simulations are performed for each configuration considering different parameters for the orbits and the results are compared in order to determine if the differences can be measured. All of the bodies are considered spherical and homogeneous.

The problem is formulated in Cartesian coordinates with the star being placed at the origin. This formulation allows us to use the same equations for both cases, thus simplifying the comparison between them. In addition to this, this formulation yields the velocities of the planet and the second body with respect to the star, which can be easily tranformed into the radial velocity.

The coordinates of the gas giant are represented by $\bar{x}$, and the coordinates of the second planet or the satellite are represented by $\bar{y}$. The corresponding equations are:

$$\ddot{\bar{x}} - (\mu_0 + \mu_1)\frac{\bar{x}}{||\bar{x}||^3} = \mu_2\left(\frac{\bar{y}-\bar{x}}{||\bar{y}-\bar{x}||^3} - \frac{\bar{y}}{||\bar{y}||^3}\right) \tag{II.20}$$

$$\ddot{\bar{y}} - (\mu_0 + \mu_2)\frac{\bar{y}}{||\bar{y}||^3} = \mu_1\left(\frac{\bar{x}-\bar{y}}{||\bar{x}-\bar{y}||^3} - \frac{\bar{x}}{||\bar{x}||^3}\right) \tag{II.21}$$

where μ_i is $G\mathcal{M}_i$, $i = 0, 1, 2$; $\mathcal{M}_0 = 1\mathcal{M}_\odot$ is the mass of the star; $\mathcal{M}_1 = 1\mathcal{M}_{Jup}$ is the mass of the planet; $\mathcal{M}_2$ is the mass of the third body; and G is the gravitational constant.

The integration will be carried out over a period of 50 years and with a step of 1 day, as we are interested in short term variations in the orbital elements, not in the long term stability of the system.

II.4.2 Method of integration

We used the TIDES package (Abad et al., 2011, 2012, 2015) for the integration of the equations of the problem which is an implementation of the Taylor series method for the resolution of Ordinary Differential Equations (ODEs). It is a robust integrator that allows multiple precision calculations and the use of variable stepsize. The package works as explained below.

We have the initial value problem:

$$\frac{d\mathbf{x}(t)}{dt} = f(t, \mathbf{x}; \mathbf{p}), \ \mathbf{x}(0) = 0, \ t \in \mathbb{R}, \mathbf{x} \in \mathbb{R}^n, \mathbf{p} \in \mathbb{R}^m \tag{II.22}$$

where $\mathbf{p}$ is a set of $m \geq 0$ parameters.

If we consider a sequence of $\{h_i\}_{i\geq 0}$ steps for each $t_{i+1} = t + h_i$ we can approach the value of $\mathbf{x}$ by its Taylor series expansion:

$$\begin{aligned} \mathbf{x}(t_{i+1}) = \mathbf{x}_{i+1} &\simeq \mathbf{x}(t_i) + \frac{d\mathbf{x}(t_i)}{dt} h_i + \ldots + \frac{1}{n!}\frac{d^n\mathbf{x}(t_i)}{dt^n} h_i^n \\ &\simeq \mathbf{x}_i + f(t_i, \mathbf{x}_i) h_i + \ldots + \frac{1}{n!}\frac{d^{n-1} f(t_i, \mathbf{x}_i)}{dt^{n-1}} h_i^n. \end{aligned} \tag{II.23}$$

The problem now is to calculate the coefficients of the series:

$$\frac{d^{n-1} f(t_i, \mathbf{x}_i)}{dt^{n-1}}.$$

This is accomplished with TIDES by using automatic differentiation techniques.

The package itself consists of two parts:

- MathTIDES, a preprocessor written in MATHEMATICA which calculates the coefficients of the series and generates the files to perform the integration, and
- LibTIDES, a C library which is used to compile and link the code generated by MathTIDES.

The package has four modes which can be selected during the preprocessor phase. There are two minimal modes the first in FORTRAN (minf-tides) and the second in C (minc-tides), and two standard modes, both in C, one working in double precision (dp-tides) and the other in multiple precision (mp-tides). TIDES uses the MPFR and GMP libraries to allow multiple precision calculation.

II.4.3 Results

As we said before, in the first scenario we consider a second planet of 1 $\mathcal{M}_{Neptune}$ and, in the second scenario, a satellite with 1 $\mathcal{M}_{\oplus}$. The second planet will be at 5.22 a.u. from the star and the satellite orbits the planet with a semimajor axis of 1 million km.

The mass of the satellite is perhaps a bit larger than one would expect from Heller and Pudritz (2015a,b) but, as it is a simulation in order to test the methodology, the increase in the expected radial veloctity signal is more informative. In addition, if the aformentioned case of Kepler-1625b is confirmed, such a large mass might not be unrealistic in some situations. For the case of the second planet, we follow a different approach and select a Neptune-sized planet instead of a gas giant because, with a larger planet, the signal would be measurable in the radial velocity of the star and there would be no necessity to search for perturbations in the signal of the first.

For the first integration, the orbits will be considered to be circular and coplanar with an inclination of 85°. Then we will increase the mutual inclination from 0° to 10° to see how this affects the signal. We will also try different eccentricities for the second body, from 0.0 to 0.5.

The outputs that we obtain from the integrator are the positions and velocities of the two bodies with respect to the star. We can calculate the elements of the osculating orbit for each planet and the satellite in order to have a clearer vision of their dynamical behavior as well as calculate the radial velocity.

II.4.4 First integration

The results of the circular, coplanar configurations show that we can indeed distinguish between the two proposed scenarios provided that we have observations covering a reasonable amount of time.

In both scenarios, we can observe that the osculating elements of the first planet present a periodic variation in the semimajor axis and in the eccentricity, with higher amplitude in the planet-satellite case (see Figures II.10 and II.11). There is also a precession in the periastron passage caused by the second planet while the satellite produces an oscillation in this orbital element. This has an effect on the radial velocity of the planet and, if we plot the difference along time, we can observe its increase (Figure II.12). The plot shows the radial velocity of the star for both cases along ten years and the difference between both cases during fifty years (the difference in the timespan is to increase the visibility of the first plot). There are two components that we can observe in this difference, both of them periodic. One component is caused by the wobble that the second planet produces in the radial velocity of the star and it has a periodicity comparable to the period of the outer orbit (approximately twelve years). The second component comes from the oscillation of the periastron of the first planet caused by the satellite and, in addition to its periodic nature, there is an increase in the amplitude of the difference as the periastron passages lose their synchronicity.

We can observe the variations that each scenario causes if we plot consecutive phases of the radial velocity curve of the star along the integration period, using the initial periastron passage and period to determine the phase. We can see a small variation in Case 1 caused by

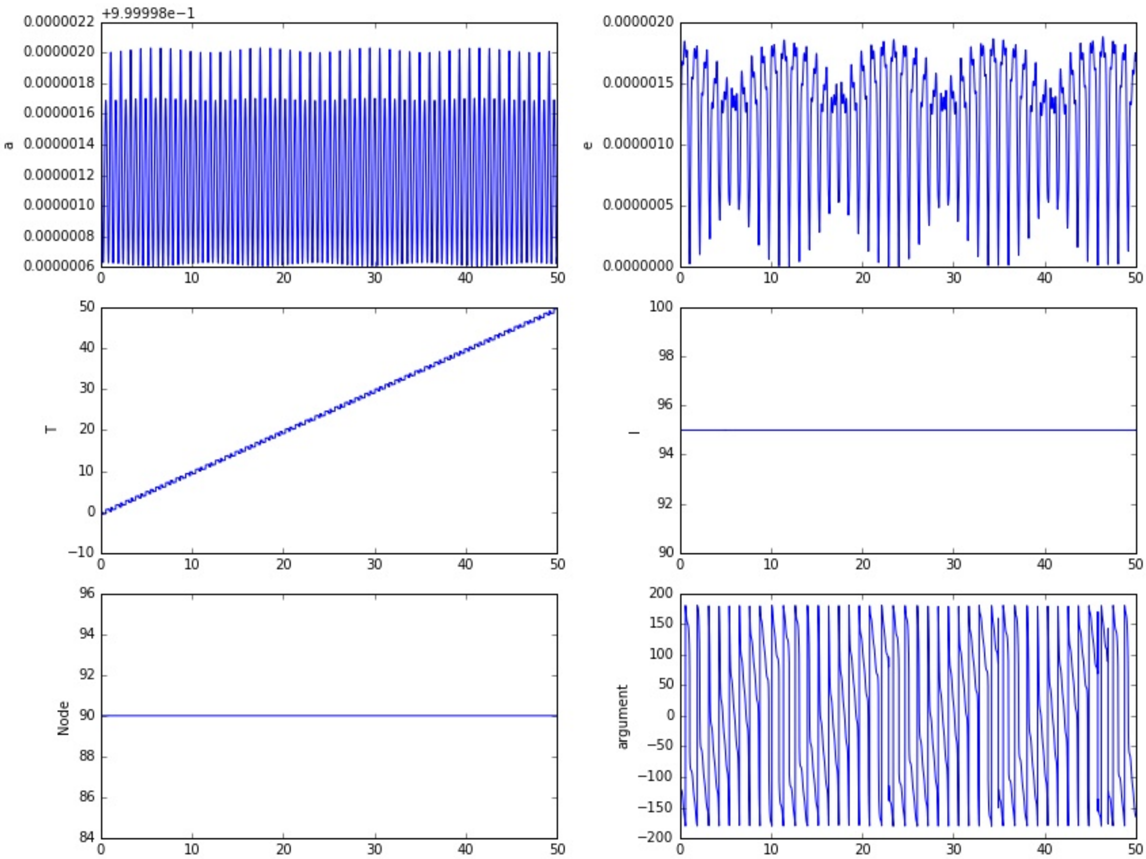

FIGURE II.10: Case 1. Variations of the orbital elements of the giant planet caused by the second planet

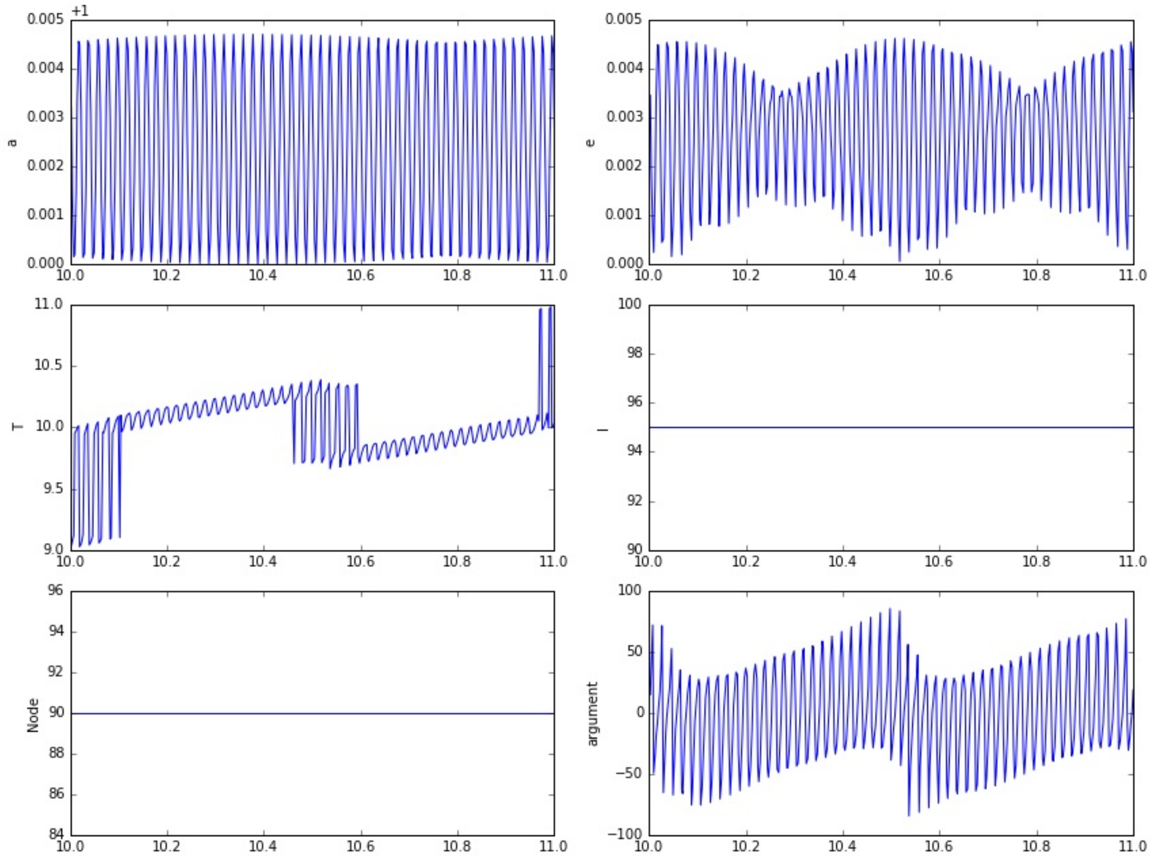

FIGURE II.11: Case 2. Variations of the orbital elements of the giant planet caused by the satellite

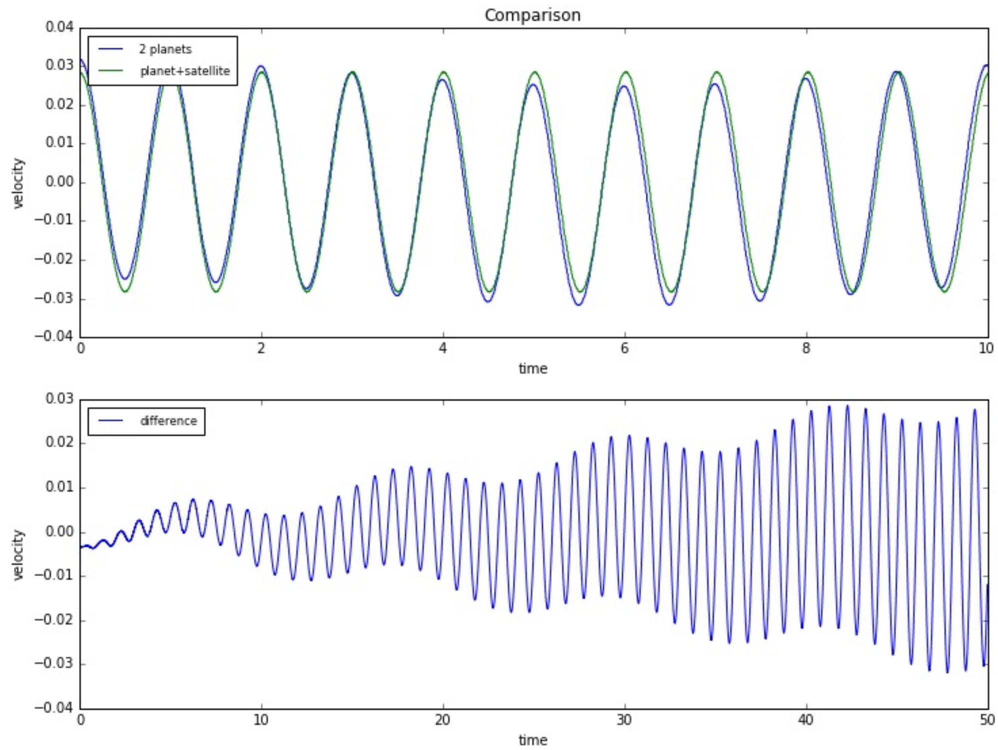

FIGURE II.12: Comparison between the radial velocities in both scenarios

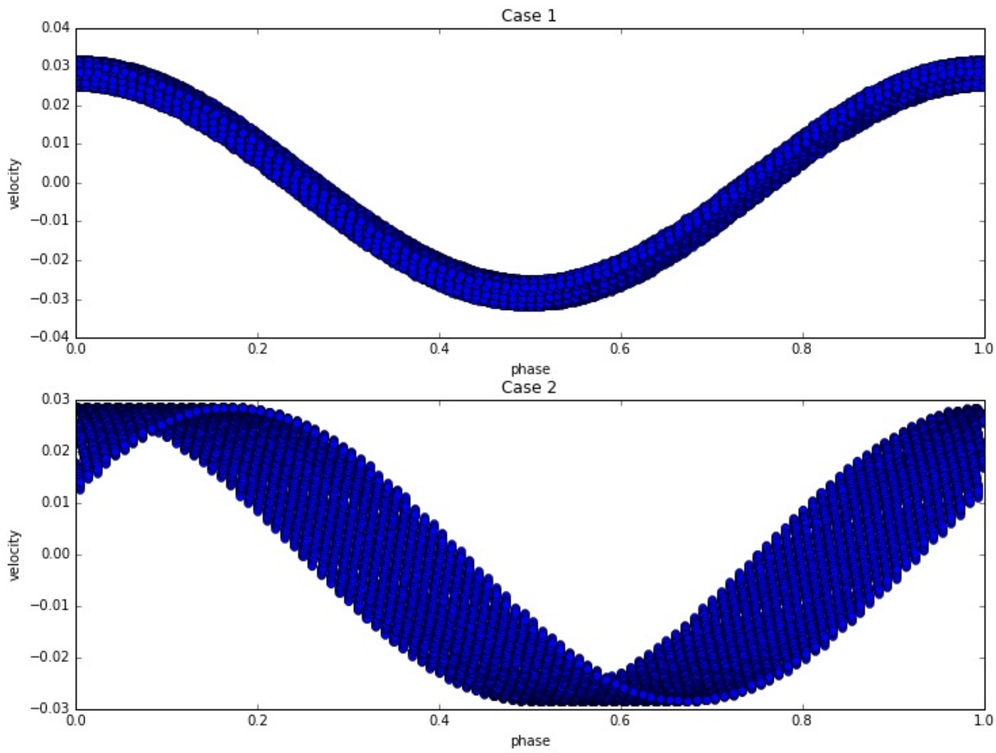

FIGURE II.13: Radial velocity curves in both scenarios

the second planet and a wide oscillation in Case 2 due to the perturbation of the periastron, allowing us to discriminate between them (Figure II.13).

II.4.5 Variations in the inclination

Now we change the inclination of the second planet and the satellite from 0° to 10° with 1° steps to see if it makes any difference.

We can see that an oscillation in the inclination of the planet appears in both scenarios, with higher amplitude and frequency in Case 2 as shown in Figures II.14 and II.15. It also causes an oscillation in the angle of the node in the case of the satellite and a slight secular increase in Case 1, at least within the short term integration. These effects increase with the mutual inclination but the effect in the radial velocities is so small that it is not noticeable (Figure II.16).

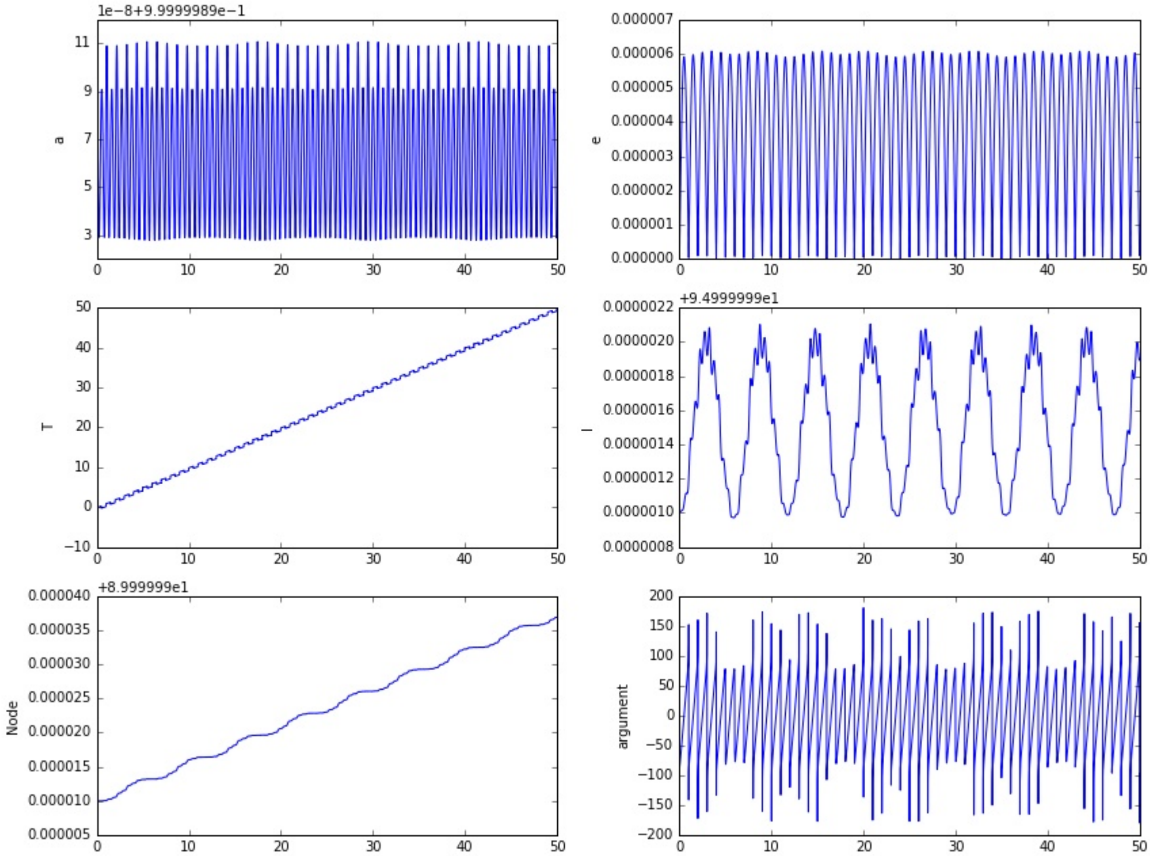

FIGURE II.14: Case 1. Variations of the orbital elements of the giant planet caused by the second planet with a mutual inclination of 5°

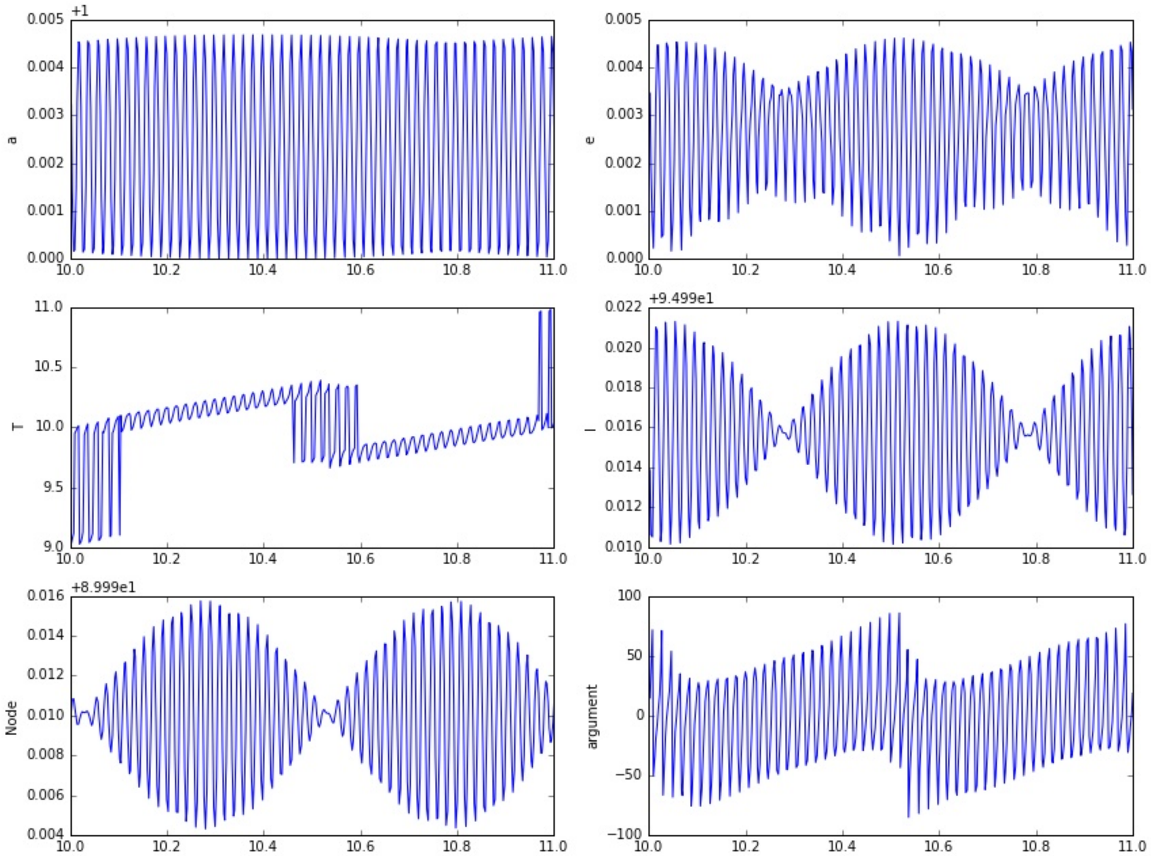

FIGURE II.15: Case 2. Variations of the orbital elements of the giant planet caused by the satellite with a mutual inclination of 5°

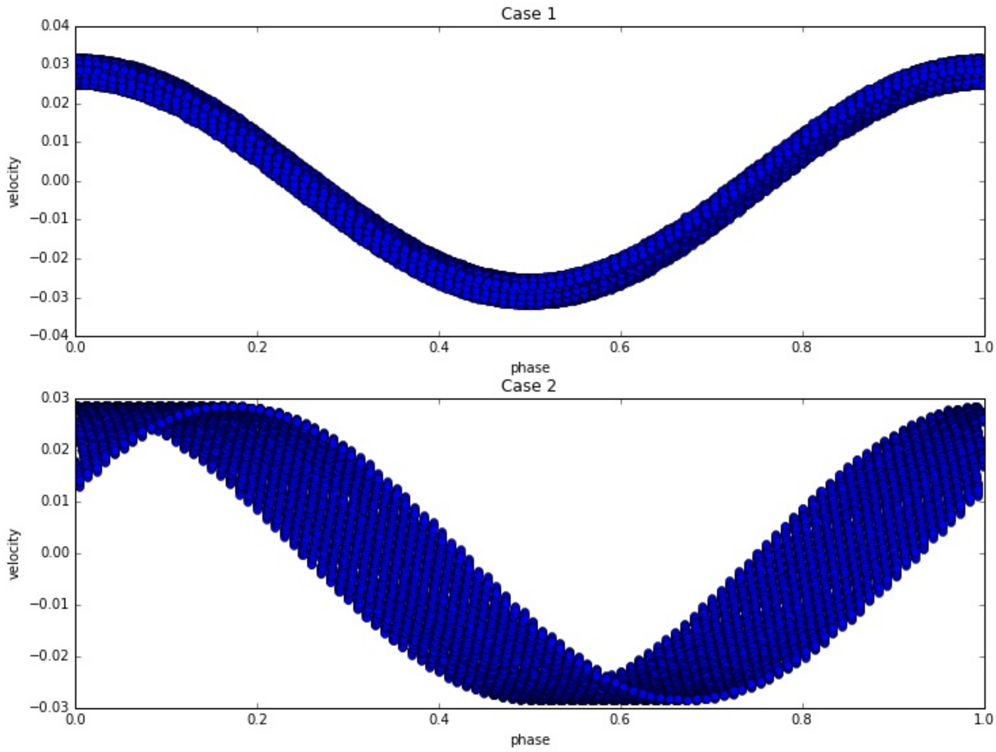

FIGURE II.16: Radial velocity curves in both scenarios with a mutual inclination of 5°

Higher mutual inclinations might cause stronger variations in the inclination of the orbit of the planet but, for planetary systems with large satellites, coplanarity or small inclinations seem to be more likely scenarios. In addition to this, high inclinations may cause instability due to the Lidov-Kozai effect (Kozai, 1962; Lidov, 1962).

II.4.6 Variations in the eccentricity

Now we will study the effect of the eccentricity. We give values from 0.0 to 0.5 (with a 0.1 step) for the eccentricity of both the satellite and the second planet.

In Case 1, the increase in the eccentricity causes a boost in the oscillation of the semimajor axis of the first planet during the closest encounter with the Neptune-size planet and an attenuation in the most distant part of the orbit (Figures II.17). The other orbital elements seem unaffected. The radial velocity of the planet doesn't show significant changes as the difference is too small.

However, the change in the eccentricity of the satellite in Case 2 produces two measurable effects on the semimajor axis perturbation. The first is an increase in the amplitude of the oscillation. The second generates a long-term perturbation superimposed over the short-term. A slight secular increase in the argument of the periastron appears which causes a boost in the observed oscillation effect as shown in Figures II.18, II.19, and II.20.

II.4.7 Conclusions

We have shown that it is possible to detect the presence of a satellite through the perturbations in the radial velocity of the host planet. These perturbations appear mainly as a variation in the periastron passage. The presence of a second planet (Case 1) causes a slow secular precession in the periastron passage while the satellite (Case 2) produces a wide oscillation in that passage. This allows us to distinguish between both scenarios provided that we have observations spanning several periods.

Regarding the orbital parameters, the main contributor to the perturbation seems to be the eccentricity whereas the mutual inclination has little effect, at least for low values.

The inclination and the angle of the node do not change in the coplanar cases, as expected. When we consider non-zero mutual inclination, we obtain a small oscillation in the inclination. This oscillation is faster and wider in Case 2 but, nevertheless, it is insufficient to cause a measurable effect in the radial velocity. The angle of the node suffers a precession effect and an oscillation in Cases 1 and 2, respectively.

A small eccentricity appears in the initially circular orbits. Both the semimajor axis and the eccentricity suffer oscillations due to the perturbation of the third body, again with greater

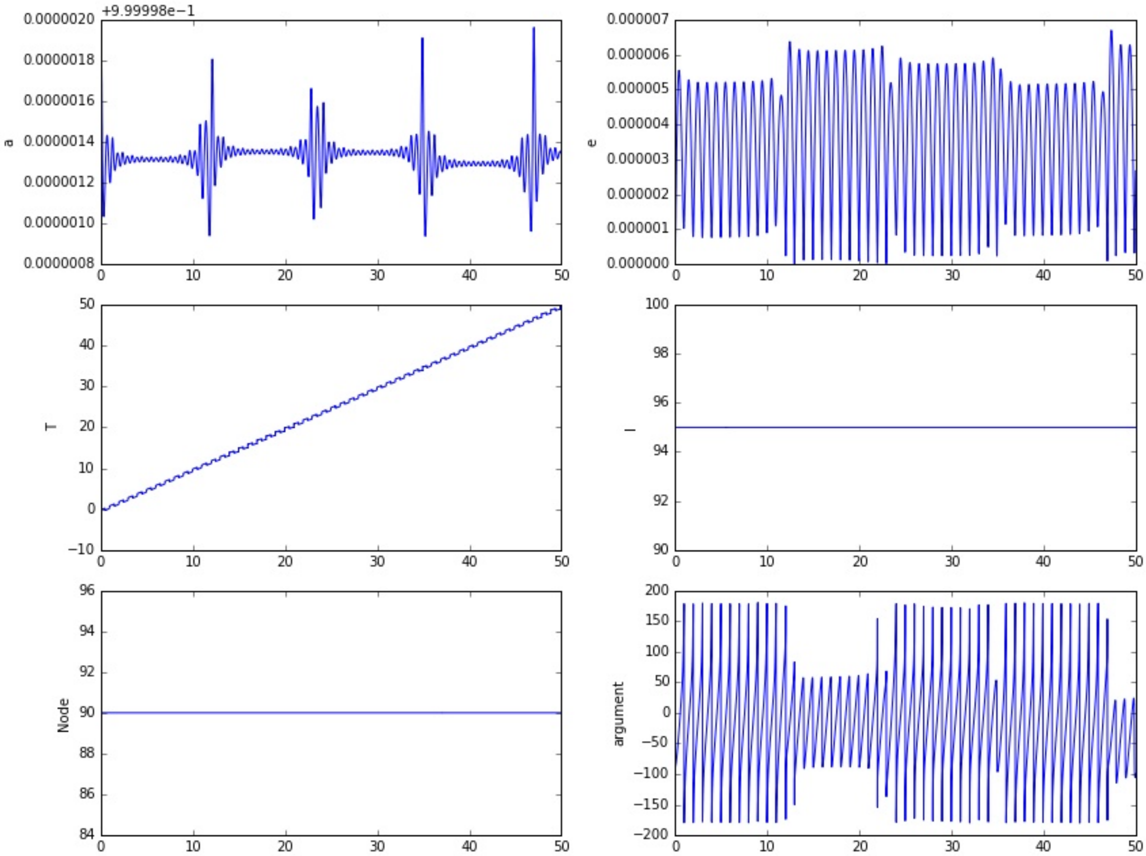

FIGURE II.17: Case 1. Variations of the orbital elements of the giant planet caused by the second planet with an orbit of eccentricity 0.5

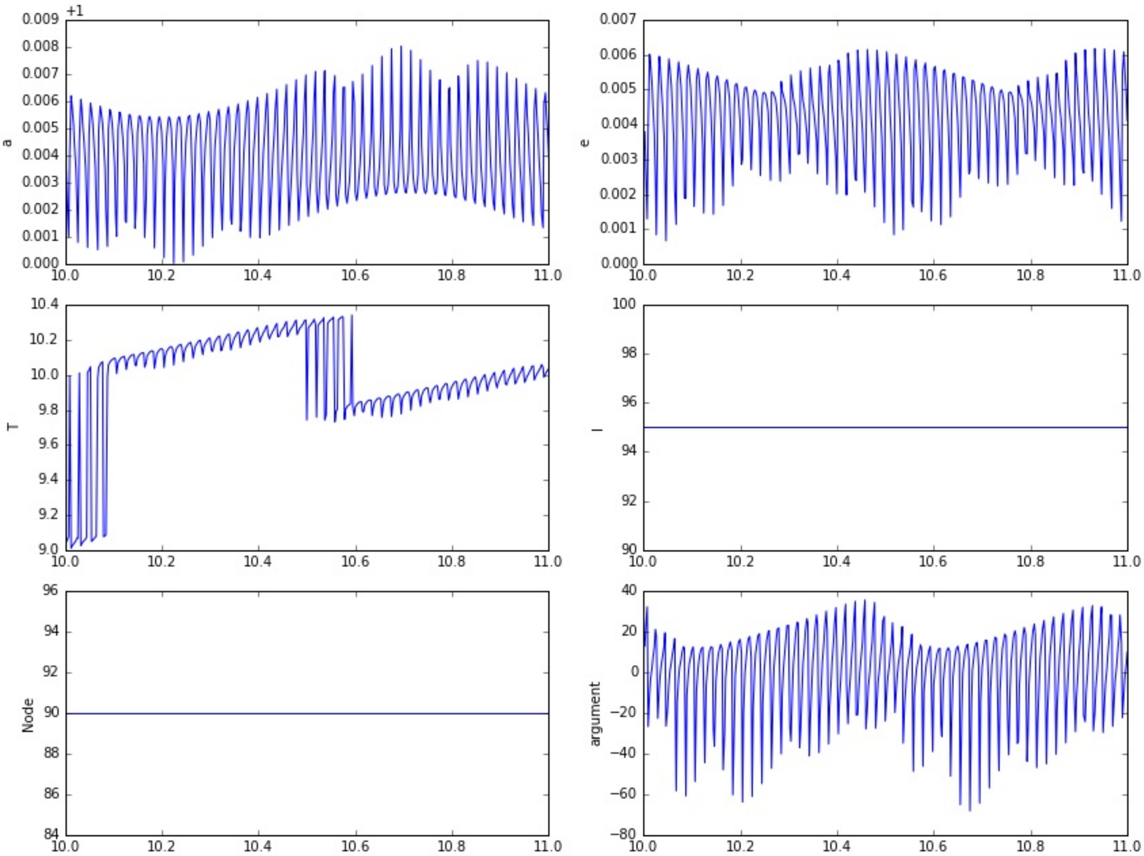

FIGURE II.18: Case 2. Short-term variations of the orbital elements of the giant planet caused by the satellite with an orbit of eccentricity 0.5

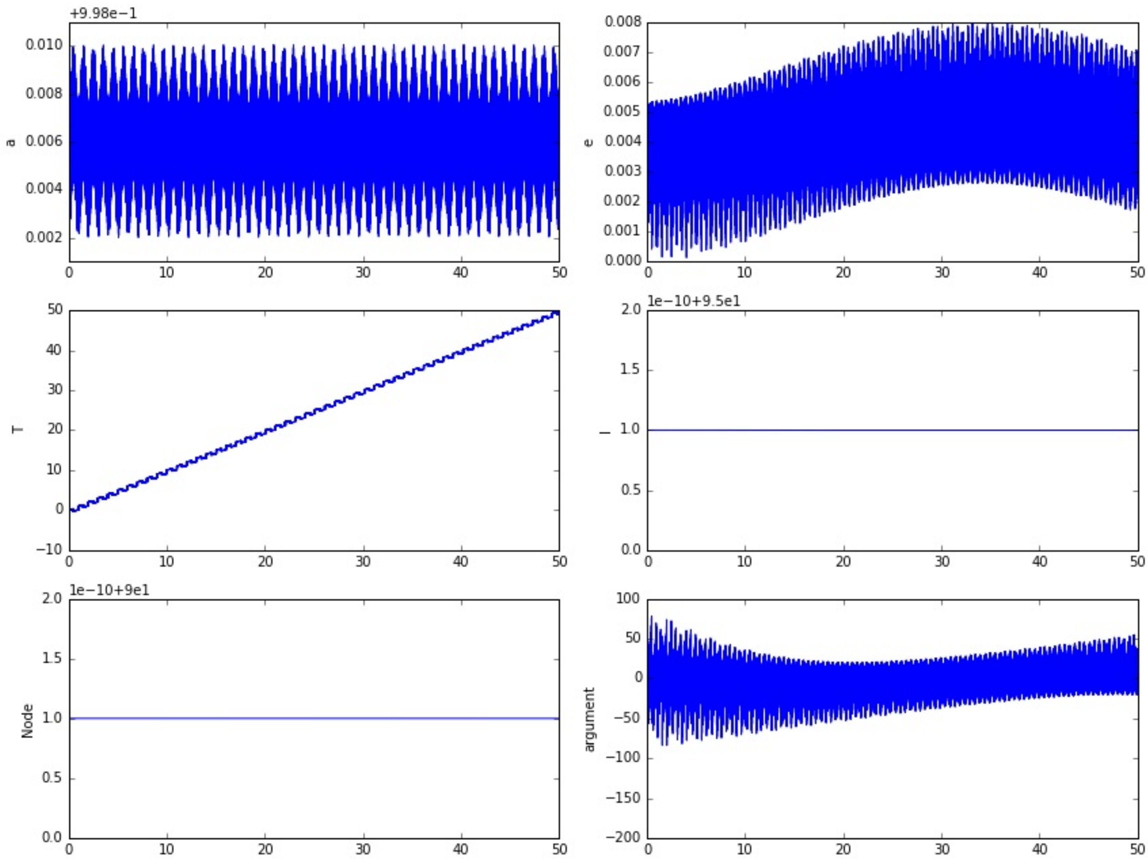

FIGURE II.19: Case 2. Mid-term variations of the orbital elements of the giant planet caused by the satellite with an orbit of eccentricity 0.5

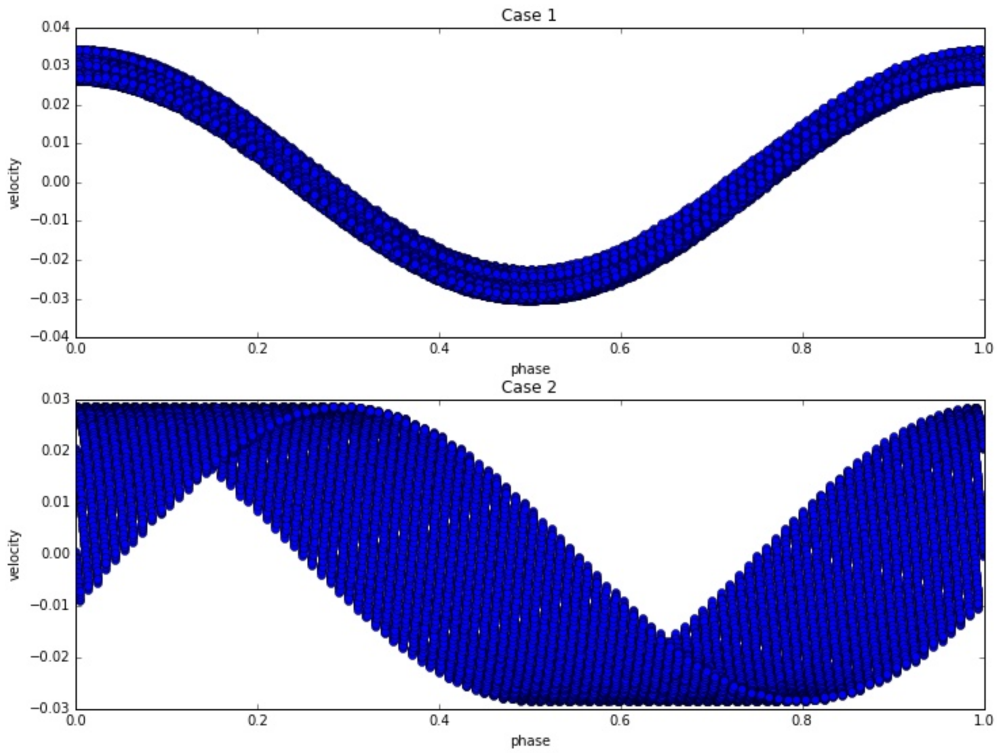

FIGURE II.20: Radial velocity curves in both scenarios with eccentricity 0.5.

amplitude and frequency in Case 2. However, the variation is not large enough to cause a measurable effect in the signal.

II.5 Habitability

One of the current goals in the study of extrasolar planets is to find planets with conditions similar to those on the Earth. That is why the concept of the Habitability Zone (HZ) has been established, i. e., the region around the star in which a planet with an atmosphere analogous to ours is able to maintain liquid water on its surface. Obviously, this only refers to habitability environments similar to our planet but other conditions could be considered. For example, a planet covered in ice with an inner ocean heated by tidal forces or by volcanic activity, as may happen within some of the satellites of our Solar System. However, it is not clear yet that these environments are able to harbor life and they would also be more difficult to detect than Earth-type planets.

Models of the atmospheres of the extrasolar planets have been formulated for the study of the habitability in late-type main-sequence stars. Those models showed how the effective temperature (T_{eff}) of the host star affects the presence of liquid water (Kasting, Whitmire, and Reynolds, 1993; Kopparapu et al., 2013a,b). Kopparapu et al. found that the stellar flux that reaches the top layer of the atmosphere can be written as a function of $T_* = T_{eff} - T_{eff\odot}$ and the flux of the Sun, $S_{eff\odot}$, in the following way:

$$S_{eff} = S_{eff\odot} + a_1 T_* + a_2 T_*^2 + a_3 T_*^3 + a_4 T_*^4. \qquad \text{(II.24)}$$

where a_1, a_2, a_3, and a_4 are parameters that depend on the limit that is chosen for the HZ. From the stellar flux, the distance is obtained by using the following expression:

$$d = (\frac{L}{S_{eff}})^{0.5} \ a.u. \qquad \text{(II.25)}$$

where L represents the luminosity of the star given in units of solar luminosity. In that work, they distinguish several lower and upper limits for habitability:

- Recent Venus. The planets located at a greater distance from the star than this limit could maintain water on their surfaces during the first millions of years after the formation of the system. However, due to the increase in the luminosity of the star when it moves away from the Zero-Age Main-Sequence point (ZAMS), the greenhouse effect on the planet increases and the liquid water on the surface evaporates.

- Runaway greenhouse effect. This is the lower limit of *current* habitability. At this distance from the star, the oceans completely evaporate due to the feedback in the greenhouse effect caused by the increasing amount of water vapor in the atmosphere.

- Moist greenhouse effect. At this limit, there is a considerable increase in the concentration of water vapor in the atmosphere due to the greenhouse effect. It is the lower limit for a continuous presence of liquid water on the surface of the planet.

- Maximum greenhouse effect. This is the outer limit of continuous habitability and it is defined by the maximum concentration of carbon dioxide in the way that it reaches its saturation in the atmosphere and, therefore, increases the greenhouse effect.

- Early Mars. This limit is based on the assumption that Mars was at a higher temperature during the first epoch of the Solar System than at present, possibly with liquid water on the surface. The explanation for this would be the presence of other greenhouse gases in the atmosphere emitted by the volcanic activity.

For each one of these limits, a different set of parameters a_1, a_2, a_3, and a_4 is obtained which defines the distance of each of them to the star. However, it has to be noted that the atmospheric models that they use are cloudless and their presence could widen the limits of the HZ (Kitzmann, 2017; Selsis et al., 2007). Besides, they only consider atmospheres composed of N_2-H_2O-CO_2 and the presence of other greenhouse effect gases such as methane may have significant effects. These gases widen the HZ in stars with temperatures over 4500 K due to the increase in the greenhouse effect and shorten it in colder stars due to the absorption of radiation in the higher layers of the atmosphere (Ramirez and Kaltenegger, 2018). It is also necessary to remark that it is not guaranteed that a planet within those limits is habitable. These values indicate a necessary, but not sufficient, condition. In order to be able to confirm the habitability, we need to study the atmosphere which would be possible during a transit by using telescopes with high sensitivity such as the James Webb Space Telescope (Barstow and Irwin, 2016; Lustig-Yaeger, Meadows, and Lincowski, 2019).

In the framework of habitability, special attention has been paid to the case of red dwarfs (Shields, Ballard, and Johnson, 2016; Wandel, 2018). First, because the surveys that have been carried out in the solar neighborhood suggest that they are the most numerous spectral type in the Galaxy. But there is also a practical reason which is that, due to their low mass and luminosity, it is easier to detect planets around them. The HZ of these stars is smaller than for other spectral types and it is much closer to the star, that is why the planets within it usually have their rotational and orbital movements coupled due to the tidal forces (Barnes, 2017; Barnes et al., 2008; Heller, Leconte, and Barnes, 2011; Kasting, Whitmire, and Reynolds, 1993). This coupling affects the climate of the planet, not only because there is a hemisphere

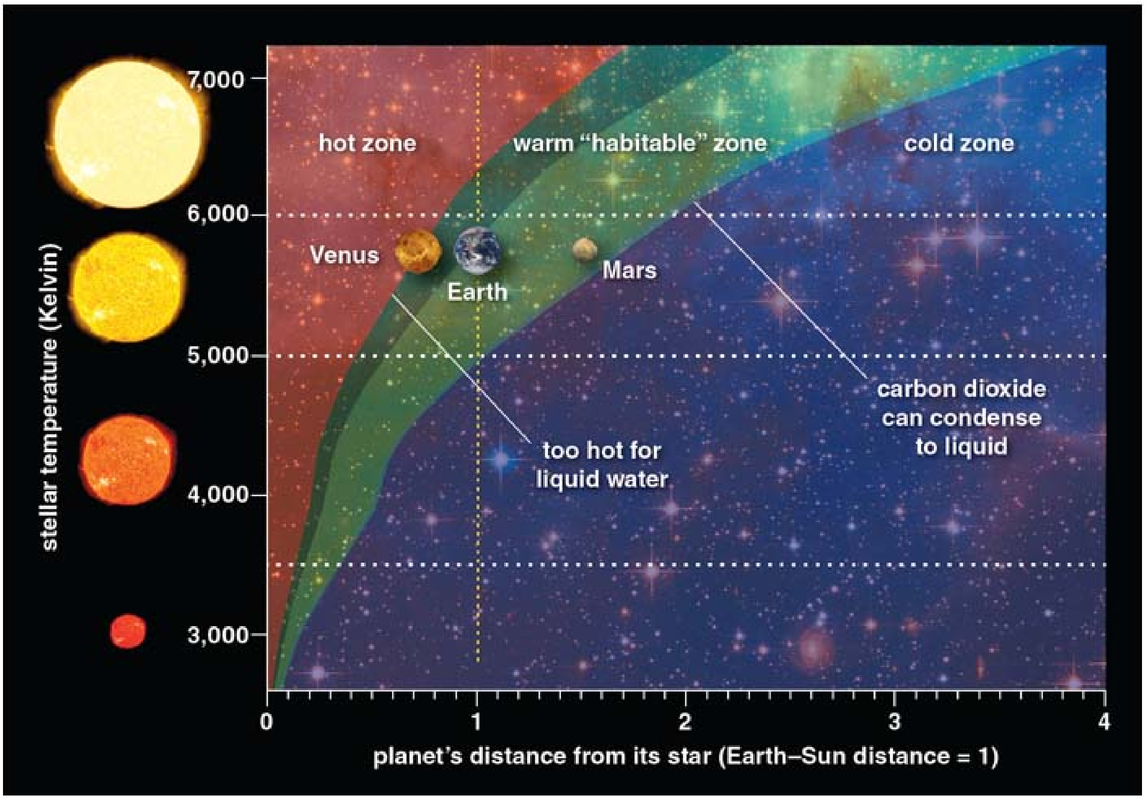

FIGURE II.21: Habitability zones depending on the distances to the star and the temperature of the star. The Recent Venus and Early Mars zones are not depicted. Image: Barbara Aulicino that is always illuminated and another that is always dark, but also because their regimes of energy circulation through the atmosphere are completely different from those of our planet (Carone et al., 2018; Lewis et al., 2018). In addition to the coupling, these type of stars have a higher rate of activity, mainly in the first years, and they produce large flares with a high emission of γ rays, X rays, and ultraviolet radiation (Loyd et al., 2019; Miles and Shkolnik, 2017; Ohm and Hoischen, 2018; Schneider and Shkolnik, 2018; Shkolnik and Barman, 2014; Wheatley et al., 2017). All of these factors lead us to think that these planets might not be habitable, mainly if their atmospheres were oxygen-poor and, therefore, lack the ability of absorbing the incoming radiation, or in planets without protective magnetic fields (Johnstone et al., 2019; O'Malley-James and Kaltenegger, 2017; Peacock et al., 2019; Tilley et al., 2019). Other authors play down the importance of the activity of these stars in the habitability, either because they consider that the inner edge of the HZ is farther away from the star than it was thought (Kopparapu et al., 2017) or due to the comparison with the UV irradiation conditions in the early phases of our planet (O'Malley-James and Kaltenegger, 2019).

The effect of the metallicity and the stellar rotation on the habitability, as well as its evolutionary state has been studied (Gallet et al., 2017). The metallicity in particular has an important influence because when it diminishes, L and T_{eff} increase. However, the rotation and the evolutionary state, as long as the star mantains in the main-sequence, barely modify the HZ. For post-main-sequence stars, it is necessary to perform a separate analysis (Ramirez and Kaltenegger, 2016). Other conditions more related to the planets have been taken into account, for example, the eccentricity of the orbit (Bolmont et al., 2016); the existence of tectonic activity due to tidal heating (Valencia, Tan, and Zajac, 2018); the lack of plate tectonics (Tosi et al., 2017); being covered by a global ocean (Kite and Ford, 2018; Levi, Sasselov, and Podolak, 2017); their magnetosphere (Blackman and Tarduno, 2018); or the obliquity of their rotation axis (Kilic, Raible, and Stocker, 2017; Nowajewski et al., 2018; Wang et al., 2016).

The study of the habitability of binary systems is also of particular interest for this Memory. Specific works have been carried out in the last years (Cuntz, 2014, 2015; Eggl et al., 2013; Eggl et al., 2012; Haghighipour and Kaltenegger, 2013; Kaltenegger and Haghighipour, 2013; Wang and Cuntz, 2019; Wang and Cuntz, 2017) that, in general, lead to several important results. In the first place, it is obviously necessary to distinguish between the cases with S-type and P-type orbits. In the S-type orbits, the HZ is mainly influenced by the star around which the planet moves but it is modified by the irradiation of the other component. In the P-type orbits, we have to consider the sum of the fluxes of both components in order to calculate the HZ. It is also important to take into account the eccentricity of the binary orbit because it affects the shape and width of the HZ to a large extent. We also have to consider the stability of the planetary orbits, as there may be regions of the HZ in which no stable orbits exist. The existence of Milankovitch cycles in binaries due to their influence in the glacial periods (Forgan, 2016) has been also studied. Finally, B. A. Wootton and R. J. Parker (Wootton and Parker, 2019) noted that binary systems located in regions with a high rate of stellar formation might have ideal conditions for habitability because the interactions with other stars in the region might move the components closer and therefore widen the HZ. However, N. Georgakarakos and S. Eggl disagree, noting that this would only be true if we consider all of the orbits to be circular (Georgakarakos and Eggl, 2019).

Regarding the exosatellites, there are also investigations concerning their habitability. Apart from the radiation incoming from the star (Heller, 2012), there are other energy sources to be taken into account in order to evaluate the habitability, e.g. the heat irradiated by the planet (Dobos, Heller, and Turner, 2017), the light of the star that it reflects (Heller et al., 2014), the synchronic rotation (Haqq-Misra and Heller, 2018), or the drop in the irradiation when the satellite is eclipsed by the planet (Heller and Barnes, 2013). However, the most important effect in the habitability of exosatellites is the tidal heating. In low mass stars with a HZ near to the star, the orbit of the satellite must be closer to the planet in order to be stable and the tidal heating is higher which may render the satellite inhabitable (Zollinger, Armstrong, and Heller, 2017). However, in solar-type stars, the conditions may be more favorable.

Chapter III

Published articles

This dissertation has been carried out in the modality of the compilation of articles published in research journals according to the normative of the *Escola de Doutoramento Internacional* of USC and the RD 99/2011. In this chapter, the published articles are presented, three of them in the journal, *Monthly Notices of the Royal Astronomical Society* (MNRAS) and another in *Astronomy Letters* (Astron. Lett.), both indexed in the *Journal Citation Report* (JCR). All were carried out under the supervision of the Director of this dissertation. Those published in MNRAS concern the study of binaries and those in Astron. Lett. Treat exoplanets and exosatellites. MNRAS is a high impact journal published by the *Oxford University Press*. They are consistently in the first quartile of JCR as it was in the years in which the articles were published, 2014 (position 12 of 60), 2017 (12/66), and 2018 (15/69). Astron. Lett. is a prestigious Russian journal of Astronomy, the international version of *Pis'ma v Astronomicheskii Zhurnal*, distributed by *Springer*. It is also indexed in the JCR. In 2014, the year of publication of the article, it was in the third quartile (39/60).

The first two articles are related to the visit to Cambridge and the associated scientific collaboration with Professor Roger F. Griffin. They deal with the study of spectro-interferometric binaries in such a way that the spectroscopic orbit as well as the visual orbit are calculated. In the first article, Elliot P. Horch of Southern Connecticut State University, is one of the authors. He contributed the new interferometric measurements obtained with the 3.5 telescope of the WIYN Observatory (Wisconsin-Indiana-Yale-NOAO) that were used for the calculation of the visual orbit. As we have seen throughout this Memory, the study of this type of binaries is of special importance. On one hand, more precise orbit calculation is permitted which is crucial when studying possible existing planetary systems. On the other hand, the orbital parallax can be determined (which yields an independent measurement of the distance and permits the calculation of the luminosity of the stars) and the individual masses of the components. The masses do not only affect the dynamic study of the planetary systems but, along with the luminosity, they influence the evolutionary development of the stars as can be seen in the second article in which is included an evolutionary study of the binary using the **PARSEC** models (`http://stev.oapd.inaf.`

it/cgi-bin/cmd). The masses and the evolutionary state determine the effective temperature and, therefore, the habitability zones around the stars. The url adrresses of the articles are https://academic.oup.com/mnras/article/444/4/3641/1027288 and https://academic.oup.com/mnras/article/469/1/1096/3605384.

In the third article, a novel methodology for the study of visual systems in which the arc of observation is small which can determine precise orbits and obtain reliable physical parameters. As indicated in the article, this is of special interest in the case of pre-main sequence stars (PMS) for which there are no good calibrations of mass. These systems can present prominent protoplanetary discs. For that reason, their study is fundamental within the frame of the evolution of planetary systems. The article can be found in https://academic.oup.com/mnras/article/476/2/2792/4840252.

The fourth article is a dynamic study of a model of a planetary system within a binary that consists of two stars with a planet orbiting one of them (orbital type S) and a satellite rotating around the planet. In the first place, the possible four body planetary systems are reviewed using the step formulation developed by A. Abad in his dissertation and classifying them according to the hierarchy that they present. Then the selected model is formulated mathematically. To that end, the Hamiltonian formulation of the system of equations is developed and resolved in an analytic manner by means of the theory of perturbations. First, a change in the Jacobi coordinates is made in order to develop the Hamiltonian in series as a function of three small parameters that are then reduced to two in order to accomplish the necessary truncation. Then the biparametric method of Hori is applied in order to resolve the system. The following link https://link.springer.com/article/10.1134%2FS1063773714110012 contains a copy of the article.

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